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Research Article

INVERSE TRANSIENT THERMOELASTIC BEHAVIOR OF ANNULAR DISC BY USING MARCHI-ZGRABLICH AND LAPLACE TRANSFORM TECHNIQUE

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ABSTRACT
The present work deals with the determination of temperature, unknown heat flux, displacement and thermal stresses of thin annular disc with the stated conditions. The inverse transient heat conduction equation with stated conditions is solved by using Marchi-Zgrablich transform and Laplace transform simultaneously and the results for temperature distribution, heat flux distribution, displacement and thermal stress functions are obtained in terms of infinite series of Bessel's function. These results solved for special case by using Mathcad 2007 software and presented graphically by using Origin software.

Key Words: *An Annular Disc, Inverse Transient Heat Conduction, Heat flux, Thermal Stresses, Marchi-Zgrablich and Laplace Integral Transform*

INTRODUCTION
The inverse heat conduction problem is one of the most frequently encountered problems by scientists. The wide varieties of problems that are covered under conduction also make it one of the most researched and thought about problems in the field of engineering and technology. This kind of problems can be solved by various methods. These inverse problems consist of determination of unknown temperature, heat flux, displacement and thermal stress functions of solids when the conditions of temperature and displacement and stress are known at the some points of the solid under consideration.
Grysa and Cialkowski (1980), Grysa and Koalowski (1982) studied one-dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. Deshmukh et.al (2010) investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by Quasi-static approach. Ghonge and Ghadle (2010) consider an inverse transient thermoelastic solid sphere and obtain the temperature, displacement and thermal stress distributions. Marchi and Zgrablich (1964) studied the heat conduction in hollow cylinder with radiation. In this work we consider thin annular disc with interior known temperature function $f(r,t)$ at some $z=\xi$. The inverse heat conduction equation is solved by using finite Marchi-Zgrablich integral transform as in Marchi and Zgrablich (1964) and Laplace transform as defined in Sneddon (1972) simultaneously and the results for temperature distribution, unknown heat flux, displacement and thermal stresses are obtained in terms of infinite series of Bessel's function and it is solved for special case by using Mathcad-2007 software and illustrated graphically by using Origin software.

THE MATHEMATICAL MODEL
Consider a thin annular disc of thickness h occupying the space D as define $D:\left\{(x,y,z)\middle| a\le r=\sqrt{x^2+y^2}\le b, 0\le z\le h\right\}$. The disc is subjected to arbitrary known interior temperature $f(r,t)$ within the region $0\le z\le h$, the more general third kind boundary conditions for temperature are maintained at zero on lower, inner and outer curved surface of annular disc. Under these

more realistic prescribed conditions, the unknown temperature on upper surface and quasi-static thermal stresses due to unknown temperature $g(r,t)$ are required to determine.

The differential equations governing the displacement function $U(r,z,t)$ as in Noda et. al (2003) is given as

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}=(1+\nu)a_t T \tag{1}$$

with $U_r=0$ at $r=a$ and $r=b$ (2)

where ν and a_t are the Poisson's ratio and linear coefficient of thermal expansion of the material of the disc and T is the temperature of the annular disc satisfying the differential equation as in Ozisik (1968)

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial T}{\partial t} \tag{3}$$

subject to the initial condition

$$T(r,z,t)\big|_{t=0}=0 \tag{4}$$

and the boundary conditions

$$\left[T(r,z,t)+k_1\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a}=0 \tag{5}$$

$$\left[T(r,z,t)+k_2\frac{\partial T(r,z,t)}{\partial r}\right]_{r=b}=0 \tag{6}$$

$$\left[T(r,z,t)+\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0}=0 \tag{7}$$

$$\left[T(r,z,t)+\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\xi}=f(r,t) \tag{8}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=h}=g(r,t) \tag{9}$$

The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr}=-2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{10}$$

$$\sigma_{\theta\theta}=-2\mu\frac{\partial^2 U}{\partial r^2} \tag{11}$$

where k is the thermal diffusivity of the material of the disc and k_1 and k_2 are the radiation constant on the curved surfaces of the disc respectively. μ is the Lame's constant, Also each of the stress functions $\sigma_{rz},\sigma_{zz},\sigma_{\theta z}$ are zero within the disc in plane state of stress.

The equations (1) to (11) constitute the mathematical formulation of the inverse transient thermoelastic problem of the annular disc.

THE SOLUTION

Result Required

Let us define finite integral transform

$$\overline{f_p}(n)=\int_a^b x f(x) S_p(k_1,k_2,\mu_n x)dx \tag{12}$$

$$f(x)=\sum_{n=1}^{\infty}\frac{\overline{f_p}(n)S_p(k_1,k_2,\mu_n x)}{C_n} \tag{13}$$

where

$$S_p(k_1,k_2,\mu_n x)=J_p(\mu_n x)\left\{G_p(k_1,\mu_n a)+G_p(k_2,\mu_n b)\right\}\\ -G_p(\mu_n x)\left\{J_p(k_1,\mu_n a)+J_p(k_2,\mu_n b)\right\}$$

$$\begin{aligned}C_n=&\frac{b^2}{2}\left\{S_p^2(k_1,k_2,\mu_n b)-S_{p-1}(k_1,k_2,\mu_n b).S_{p+1}(k_1,k_2,\mu_n b)\right\}\\&-\frac{a^2}{2}\left\{S_p^2(k_1,k_2,\mu_n a)-S_{p-1}(k_1,k_2,\mu_n a).S_{p+1}(k_1,k_2,\mu_n a)\right\}\end{aligned} \tag{14}$$

Equations (12) and (13) define the finite Marchi-Zgrablich integral transform of order p and its inverse transform respectively. Further we include an operational property

$$\begin{aligned}\int_a^b x\left\{\frac{\partial^2 f}{\partial x^2}+\frac{1}{x}\frac{\partial f}{\partial x}+\frac{p^2 f}{x^2}\right\}S_p^2(k_1,k_2,\mu_n x)=&\frac{b}{k_2}S_p^2(k_1,k_2,\mu_n b)\left\{f+k_2\frac{\partial f}{\partial x}\right\}_{x=b}\\&-\frac{a}{k_1}S_p^2(k_1,k_2,\mu_n a)\left\{f+k_1\frac{\partial f}{\partial x}\right\}_{x=a}-\mu_n^2\overline{f_p}(n)\end{aligned} \tag{15}$$

Determination Temperature Function $T(r,z,t)$

By applying first finite Marchi-Zgrablich integral transform as define in equation (12) to the equations (3), (4), (7),(8) and using (5), (6) to reduce differential equation in Marchi-Zgrablich transform domain then applying Laplace transform as in Sneddon (1972) and making use respective inversion transforms as in (13) and Sneddon (1972), once we get the temperature function as

$$\begin{aligned}T(r,z,t)=2k\pi\sum_{n=1}^{\infty}\frac{1}{C_n}&\left\{\sum_{m=1}^{\infty}\frac{m(-1)^m}{(\xi^2+m^2\pi^2)}\left[m\pi\cos\left(\frac{m\pi z}{\xi}\right)-\xi\sin\left(\frac{m\pi z}{\xi}\right)\right]\right.\\&\left.\times\int_0^t\overline{f}(n,t').\exp\left[-k\left(\mu_n^2+\frac{m^2\pi^2}{\xi^2}\right)(t-t')\right]dt'\right\}S_0(k_1,k_2,\mu_n r)\end{aligned} \tag{16}$$

where

$$S_0(k_1,k_2,\mu_n x)=J_0(\mu_n x)\left\{G_0(k_1,\mu_n a)+G_0(k_2,\mu_n b)\right\}\\ -G_0(\mu_n x)\left\{J_0(k_1,\mu_n a)+J_0(k_2,\mu_n b)\right\} \tag{17}$$

Determination of Heat Flux Distribution $g(r,t)$ ***on Upper Plan Surface***

substituting the value of $T(r,z,t)$ from equation (17) in equation (9) one obtain the heat flux distribution $g(r,t)$ on upper plan surface as

$$g(r,t) = 2k\pi \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{m(-1)^{m+1}}{(\xi^2 + m^2\pi^2)} \left[\frac{m^2\pi^2}{\xi} \sin\left(\frac{m\pi h}{\xi}\right) - m\pi \cos\left(\frac{m\pi h}{\xi}\right) \right] \right.$$
$$\left. \times \int_0^t \overline{f}(n,t').\exp\left[-k\left(\mu_n^2 + \frac{m^2\pi^2}{\xi^2} \right)(t-t') \right] dt' \right\} S_0(k_1,k_2,\mu_n r) \tag{19}$$

Determination of Displacement Function $U(r,z,t)$

substituting the value of $T(r,z,t)$ from equation (17) in equation (1) one obtain the thermoelastic displacement function $U(r,z,t)$ as,

$$U(r,z,t) = 2k\pi(1+\nu)a_t \sum_{n=1}^{\infty} \frac{1}{C_n \mu_n^2} \left\{ \sum_{m=1}^{\infty} \frac{m(-1)^m}{(\xi^2 + m^2\pi^2)} \left[m\pi \cos\left(\frac{m\pi z}{\xi}\right) - \xi \sin\left(\frac{m\pi z}{\xi}\right) \right] \right.$$
$$\left. \times \int_0^t \overline{f}(n,t').\exp\left[-k\left(\mu_n^2 + \frac{m^2\pi^2}{\xi^2} \right)(t-t') \right] dt' \right\} S_0(k_1,k_2,\mu_n r) \tag{20}$$

Determination of Stress Functions σ_{rr} ***and*** $\sigma_{\theta\theta}$

Using the equation (20) in equations (10) and (11) one obtain the stress functions σ_{rr} and $\sigma_{\theta\theta}$

$$\sigma_{rr} = -4k\pi(1+\nu)a_t\mu \sum_{n=1}^{\infty} \frac{1}{r.C_n.\mu_n^2} \left\{ \sum_{m=1}^{\infty} \frac{m(-1)^m}{(\xi^2 + m^2\pi^2)} \left[m\pi \cos\left(\frac{m\pi z}{\xi}\right) - \xi \sin\left(\frac{m\pi z}{\xi}\right) \right] \right.$$
$$\left. \times \int_0^t \overline{f}(n,t').\exp\left[-k\left(\mu_n^2 + \frac{m^2\pi^2}{\xi^2} \right)(t-t') \right] dt' \right\} S_0'(k_1,k_2,\mu_n r) \tag{21}$$

$$\sigma_{\theta\theta} = -4k\pi(1+\nu)a_t\mu \sum_{n=1}^{\infty} \frac{1}{C_n} \left\{ \sum_{m=1}^{\infty} \frac{m(-1)^m}{(\xi^2 + m^2\pi^2)} \left[m\pi \cos\left(\frac{m\pi z}{\xi}\right) - \xi \sin\left(\frac{m\pi z}{\xi}\right) \right] \right.$$
$$\left. \times \int_0^t \overline{f}(n,t').\exp\left[-k\left(\mu_n^2 + \frac{m^2\pi^2}{\xi^2} \right)(t-t') \right] dt' \right\} S_0''(k_1,k_2,\mu_n r) \tag{22}$$

SPECIAL CASE AND DISCUSSION

For formulation of special case and examine the numerical calculation of analytical behavior of an annular disc, we consider the following functions and parameters:

Set $f(r,t) = (1-e^t)\delta(r), t = 1\ \text{sec}$,

Inner radius of an annular disc $a = 1m$, Outer radius of an annular disc $b = 2m$,

Thickness of an annular disc $h = 0.2m$, $\xi = 0.1m$.

We set for our convenience, $A = 2k\pi, B = 2k\pi(1+\nu)a_t$ and $C = -4k\pi(1+\nu)a_t\mu$ which assume to be constants.

The numerical calculation has been carried out with the help of computational mathematical software Mathcad-2007 and graphs are presented here using Origin software.

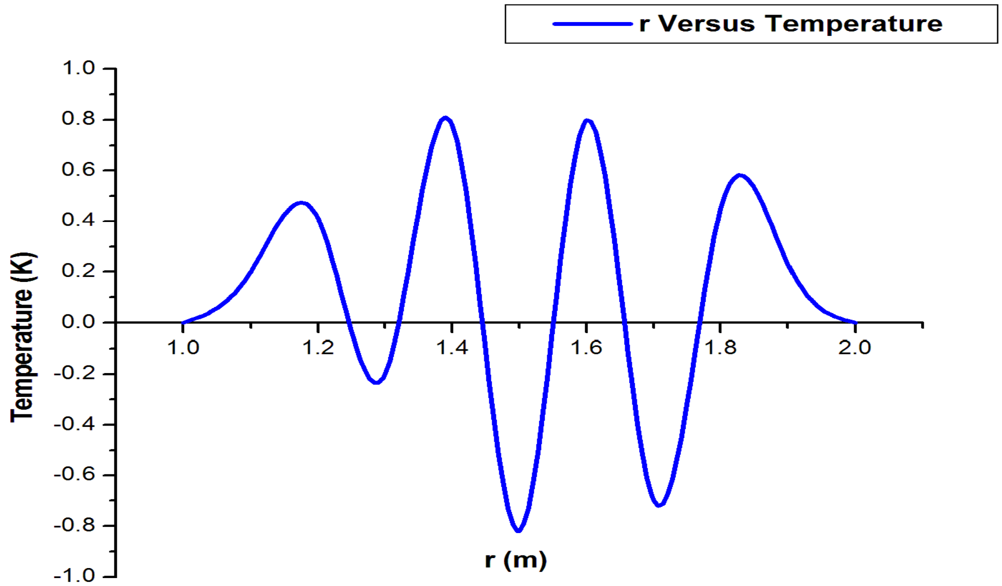

Figure 1: Show the temperature distribution in an annular disc.

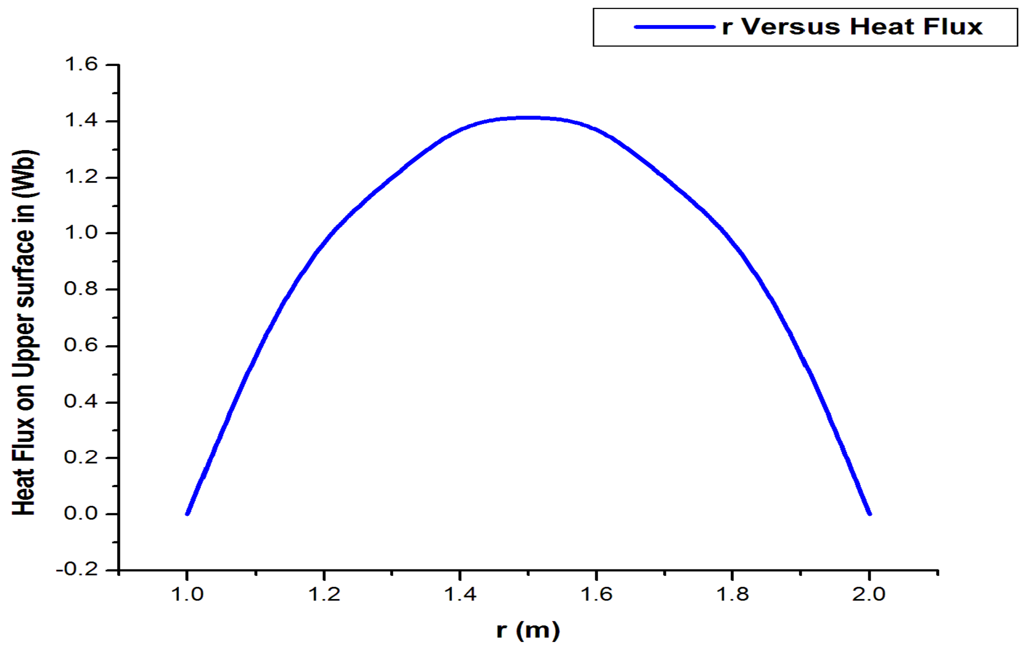

Figure 2: Show the heat flux distribution on upper surface of an annular disc.

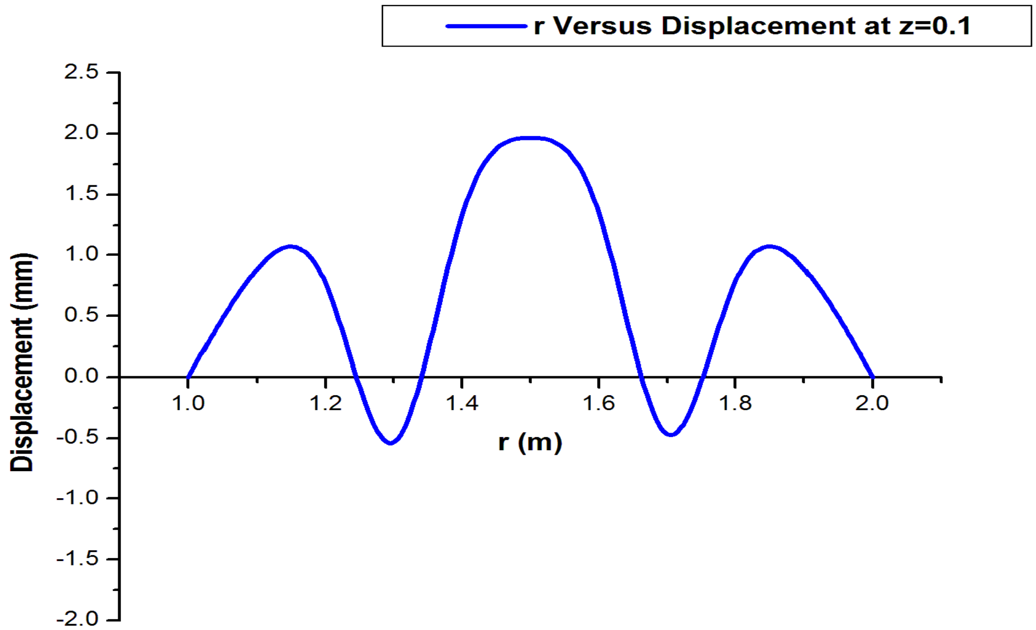

Figure 3: Show the displacement distribution in annular disc

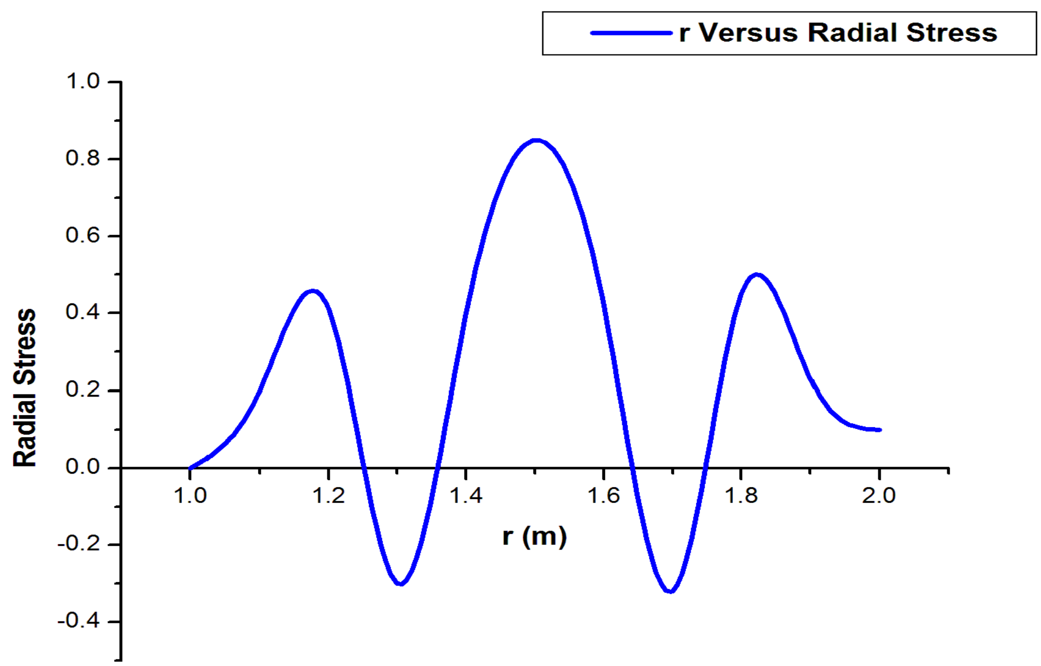

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Figure 4: Show the radial stress distribution in an annular disc.

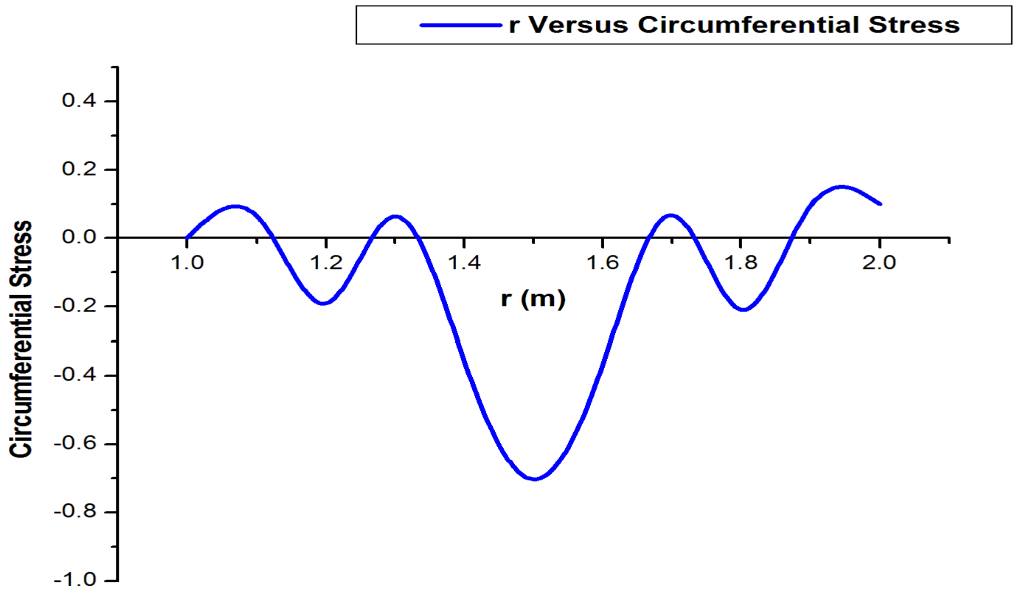

Figure 5: Show the circumferential stress distribution in an annular disc.

From Figure 1 it is observe that the temperature in analytically distribute in the interior of disc, which results in rise to displacement in the annular disc.
Figure 2 show that the heat flux on upper surface of disc attains maximum magnitude due to the temperature effect in the disc.
Figure 3 show that the displacement occurs at the middle part of the disc and analytically vanishes on either boundary surfaces of an annular disc.
Figure 4 and Figure 5 represents that the radial and circumferential stresses are develops in an annular disc respectively, and spread analytically.

Conclusion

In this work we construct the problem to determine the temperature, unknown heat flux distribution on upper surface, displacement distribution and thermal stress distribution due to interior temperature flow in the annular disc. To solve the inverse transient heat conduction equation we develop the finite Marchi-Zgrablich transform and Laplace transform simultaneously, and the results here obtained in the form of series of Bessel's functions. This solution are definitely converges because it is bounded in the finite range. As a special case and numerical results the functions and parameters are consider and the temperature, heat flux on upper surface of an annular disc, displacement and thermal stresses are determine by using Mathcad software 2007 and illustrated graphically by using Origin software. This type of inverse problems has the many applications in engineering such as main shaft of a lathe machine and aircraft structure. The results obtained here are mainly useful in the determination of state of strain in an annular disc.

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