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Research Article

MHD FLOW OF A COUPLE STRESS FLUID THROUGH A POROUS MEDIUM IN A PARALLEL PLATE CHANNEL IN PRESENCE OF EFFECT OF INCLIENED MAGNETIC FIELD

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ABSTRACT
In this paper we discuss the steady hydro magnetic flow of a couple stress fluid in a parallel plate channel through a porous medium under the influence of a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries. The perturbations are created by a constant pressure gradient along the plates. The equations for the couple stress fluid flow in the porous medium are based on Brinkman's model. The exact solution of the velocity in the porous medium is analytically derived, its behaviour computationally discussed with reference to the various governing parameters. The shear stresses on the boundaries and the discharge between the plates are also obtained analytically and their behaviour is computationally discussed.

Key Words: *Couple Stress Fluid, Porous Medium, Inclined Magnetic Field, MHD Flows*

INTRODUCTION
The MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Currently, MHD effects are widely exploited in different industrial processes ranging from metallurgy to the production pure crystals. A field in which MHD will play an essential role is nuclear fusion, where it is involved in at least two different problems: the confinement and dynamics of plasma, and the behaviour of the liquid metal alloys employed in some of the currently considered designs of tritium breeding blankets. In recently years the hydro magnetic flow in a rotating channel in the presence of an applied uniform magnetic field as well as constant pressure gradient has been considered by a number of research workers, taking into account the various aspects of the problem.

A fluid flow driven by a pulsatile pressure gradient through porous media is of great interest in physiology and biomedical engineering. Such a study has application in the dialysis of blood through artificial kidneys or blood flow in the lung alveolar sheet. Ahmadi and Manvi (1971) derived a general equation of motion for flow through porous medium and applied it to some fundamental flow problems. Rapits[8] has studied the flow of a polar fluid through a porous medium, taking angular velocity into account. The problem of peristaltic transport in a cylindrical tube through a porous medium has been investigated by El-Shehawey and El-Sebaei (2000), their results show that the fluid phase means axial velocity increases with increasing the permeability parameter. Afifi and Gad (2001) have studied the flow of a Newtonian, incompressible fluid under the effect of transverse magnetic field through a porous medium between infinite parallel walls on which a sinusoidal traveling wave is imposed. The flow characteristics of a Casson fluid in a tube filled with a homogenous porous medium was investigated by Dash *et.al* (1996). Bhuyan Hazarika (2001) has studied the pulsatile flow of blood in a porous channel in the presence of transverse magnetic field. The flows in bends and branches are of interest in a physiological context for several reasons. The additional energy losses due to the local disturbances of the flow are of interest in calculating the air flow in the lungs and in wave-propagation models of the arterial system. The details of the pressure and shear stress distribution on the walls of a bend or bifurcation are of interest in the study of parthenogenesis because it appears that the localization of plaques is related to the local flow patterns. In vascular surgery questions arise, such as what is the best angle for vascular graft to enter an existing artery in a coronary bypass (Skalak and Ozkaya, 2000). The theory of laminar, steady one-dimensional gravity flow of a non-Newtonian fluid along a solid plane surface for a fluid exhibiting slope at the wall has been studied by Astarita *et al.*, (1964). Suzuki and Tanaka (1971) have carried out some experiments on non-Newtonian fluid along an inclined plane. The flow of Rivlin-Ericksen incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field has been studied by Rathod and Shrikanth (1998). Rathod and Shrikanth (1998) have studied the MHD flow of Rivlin-Ericksen fluid between two infinite parallel inclined plates. The gravity flow of a fluid with couple stress along an inclined plane at an angle with horizontal has been studied by Chaturani and Upadhya (1977). Rathod and Thippeswamy (1999) have studied the pulsatile flow of blood through a closed rectangular channel in the presence of micro-organisms for gravity flow along an inclined channel. Hence, it appears that inclined plane is a useful device to study some properties of non-Newtonian fluids. In this paper we discuss the steady hydro magnetic flow of a couple stress fluid in a parallel plate channel through a porous medium under the influence of a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries.

FORMULATION AND SOLUTION OF THE PROBLEM

We consider an incompressible viscous and electrically conducting couple stress fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries in the transverse *xy*-plane. We choose a Cartesian system O(*x, y, z*) such that the boundary walls are at $z=0$ and $z=l$ and are assumed to be parallel to *xy*-plane. The steady flow through porous medium is governed by Brinkman's equations. At the interface the fluid satisfies the continuity condition of velocity and stress. The boundary plates are assumed to be parallel to *xy*-plane and the magnetic field of strength H_o inclined at an angle of inclination α to the *z*-axis in the transverse *xz*-plane. The component along *z*-direction induces a secondary flow in that direction while its *x*-components changes perturbation to the axial flow. The steady hydro magnetic equations governing the couple stress fluid under the influence of a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with reference to a frame are

$$\frac{\eta}{\rho}\frac{d^4u}{dz^4} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2u}{dz^2} - \frac{\sigma\mu_e^2 H_0^2 Sin^2\alpha}{\rho}u - \frac{\nu}{k}u \tag{2.1}$$

$$\frac{\eta}{\rho}\frac{d^4w}{dz^4} = \nu\frac{d^2w}{dz^2} - \frac{\sigma\mu_e^2 H_0^2 Sin^2\alpha}{\rho}w - \frac{\nu}{k}w \tag{2.2}$$

Where, (u, w) are the velocity components along O(*x, z*) directions respectively. ρ is the density of the fluid, μ_e is the magnetic permeability, ν is the coefficient of kinematic viscosity, k is the permeability of the medium, H_o is the applied magnetic field.

Let $q = u + iw$

Now combining the equations (2.1) and (2.2), we obtain

$$\frac{\eta}{\rho}\frac{d^4q}{dz^4} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2q}{dz^2} - \frac{\sigma\mu_e^2 H_0^2 Sin^2\alpha}{\rho}q - \frac{\nu}{k}q \tag{2.3}$$

The boundary conditions are

$$q = 0 \quad , \quad at \quad z = 0 \tag{2.4}$$

$$q = 0 \quad , \quad at \quad z = l \tag{2.5}$$

$$\frac{d^2q}{dz^2}=0, \qquad at \qquad z=0 \tag{2.6}$$

$$\frac{d^2q}{dz^2}=0, \qquad at \qquad z=l \tag{2.7}$$

We introduce the non-dimensional variables

$$z^*=\frac{z}{l}, \quad q^*=\frac{ql}{\nu}, \quad p^*=\frac{pl^2}{\rho\nu^2}$$

Using the non-dimensional variables, the governing non-dimensional equations are (dropping asterisks)

$$S\frac{d^4q}{dz^4}-\frac{d^2q}{dz^2}+\left(M^2Sin^2\alpha+D^{-1}\right)q=P \tag{2.8}$$

where ,

$M^2=\dfrac{\sigma\mu_e^2H_0^2l^2}{\rho\nu}$ is the Hartmann number,

$D^{-1}=\dfrac{l^2}{k}$ is the inverse Darcy parameter,

$S=\dfrac{\eta}{\rho l^2\nu}$ is the Couple stress parameter ,

$P=-\dfrac{\partial p}{\partial x}$ is the imposed pressure gradient.

Corresponding boundary conditions are

$$q=0, \qquad at \qquad z=0 \tag{2.9}$$

$$q=0, \qquad at \qquad z=1 \tag{2.10}$$

$$\frac{d^2q}{dz^2}=0, \qquad at \qquad z=0 \tag{2.11}$$

$$\frac{d^2q}{dz^2}=0, \qquad at \qquad z=1 \tag{2.12}$$

Solving the equation (2.8) making use of the boundary conditions from (2.9) to (2.12), we obtain

$$q=Ae^{m_1z}+Be^{m_2z}+Ce^{-m_1z}+De^{-m_2z}+\frac{P}{M^2Sin^2\alpha+D^{-1}} \tag{2.13}$$

The shear stresses on the upper plate and lower plate are given by

$$\tau_U=\left(\frac{dq}{dz}\right)_{z=1} \quad and \quad \tau_L=\left(\frac{dq}{dz}\right)_{z=0}$$

Where, the constants *A, B, C* and *D* are mentioned in appendix.

RESULTS AND DISCUSSION

The velocities representing the ultimate flow have been computed numerically for different sets of governing parameters namely viz. The Hartmann parameter M, the inverse Darcy parameter D^{-1} and couple stress parameter S and their profiles are plotted in figures (Fig. 1-3) and (Fig. 4-6) for the velocity components u and v respectively. For computational purpose we have assumed an angle of inclination α and the applied pressure gradient in the x-direction and are fixed. Since the thermal buoyancy balances the pressure gradient in the absence of any other applied force in the direction, the flow takes place in planes parallel to the boundary plates. However the flow is three dimensional and all the perturbed variables have been obtained using boundary layer type equations, which reduce to two coupled differential equations for a complex velocity.

We notice that the magnitude of the velocity component u reduces and v increases with increasing the intensity of the magnetic field M, the other parameters being fixed, it is interesting to note that the resultant velocity experiences retardation with increasing M (Figs. 1 & 4). (Figs. 2 & 5) exhibit both the velocity components u and v reduces with increasing the inverse Darcy parameter D^{-1}. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity experiences retardation with increasing the inverse Darcy parameter D^{-1}. Here we observe that the retardation due to an increase in the porous parameter is more rapid than that due to increase in the Hartmann number M. In other words, the resistance offered by the porosity of the medium is much more than the resistance due to the magnetic lines of force. We notice that u exhibits a great enhancement in contrast to v which retards appreciably with increase in the couple stress parameter S, but the resultant velocity shows and appreciable enhancement with in S (Fig. 3 & 6).

The shear stresses on the upper and lower plates and the discharge between the plates are calculated computationally and tabulated in the tables (1-5). The magnitude of these stresses at the upper plate is significantly high compared to the respective magnitudes at the lower plate. We notice that the magnitude of the both stresses τ_x and τ_y reduces in the upper plate and lower plates with increasing M and D^{-1}, while on the upper plate τ_x rapidly enhances and τ_y reduces and vice versa in the lower plate with increase in the couple stress parameter S. The retardation at the upper plate is significantly low compared to enhancement at the lower plate (Tables. 1-4). The discharge Q reduces in general with increase in the intensity of the magnetic field M and lower permeability of the porous medium (corresponding to an increase in D^{-1}) and enhances the couple stress parameter S (Table. 5).

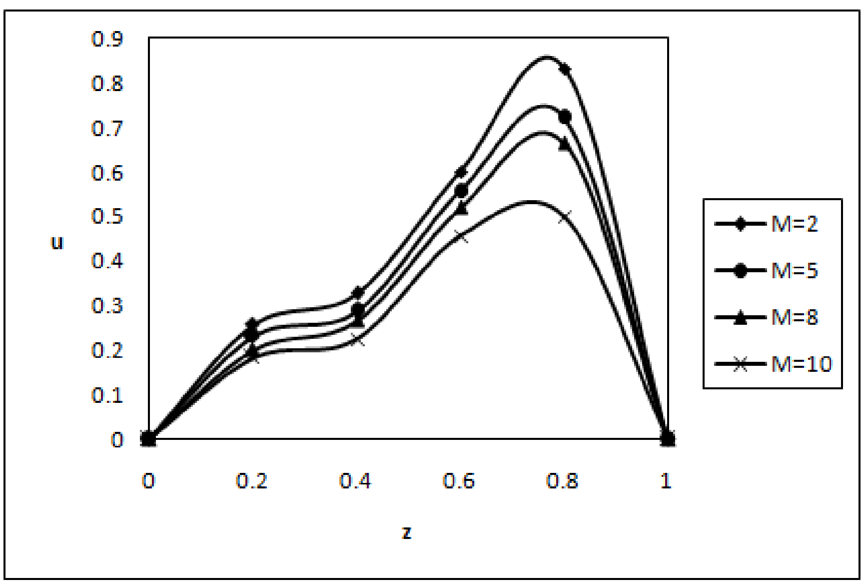

Figure 1: The velocity profile u for different M with $D^{-1}=1000, S=1, \alpha=\frac{\pi}{4}$

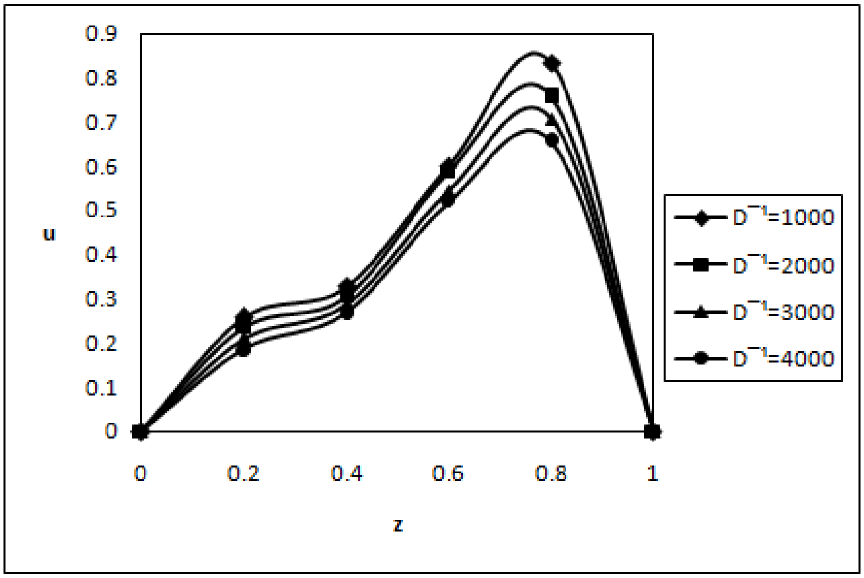

Figure 2: The velocity profile u for different D^{-1} with $M=2$, $S=1$, $\alpha = \frac{\pi}{4}$

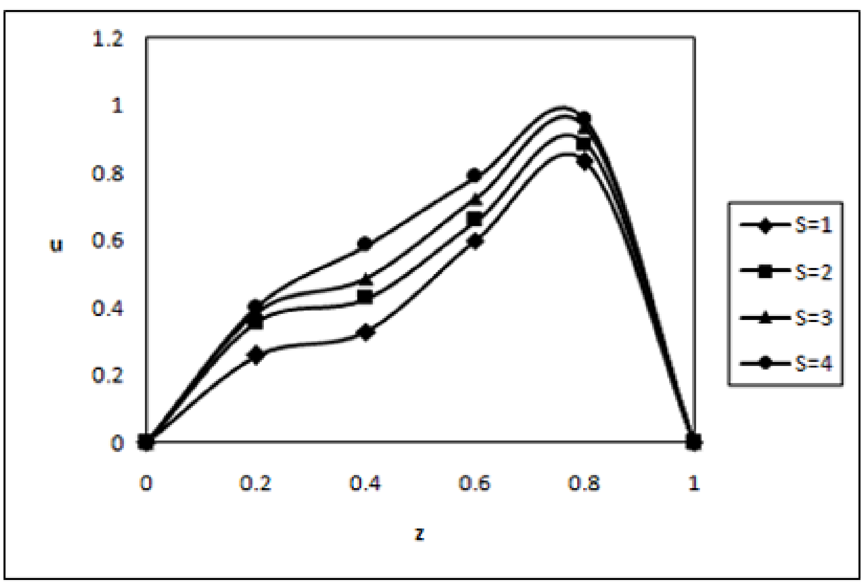

Figure 3: The velocity profile u for different S with $D^{-1}=1000$, $M=2$, $\alpha = \frac{\pi}{4}$

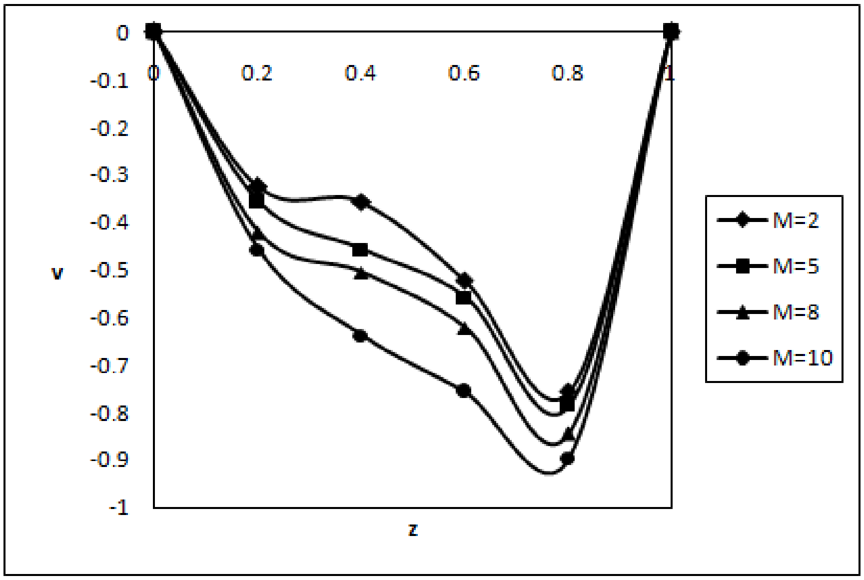

Figure 4: The velocity profile *v* for different *M* with D^{-1}*=1000, S=1,* $\alpha = \frac{\pi}{4}$

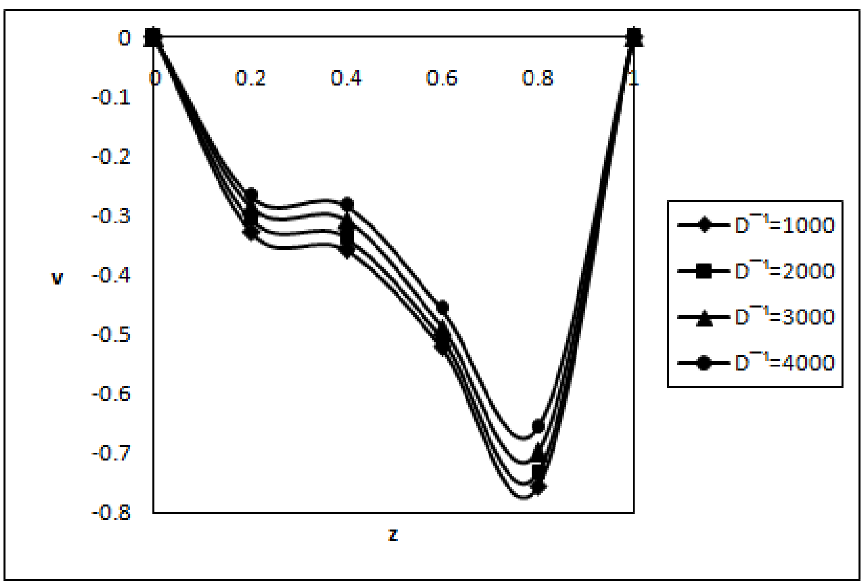

Figure 5: The velocity profile *v* for different D^{-1} with *M=2, S=1,* $\alpha = \frac{\pi}{4}$

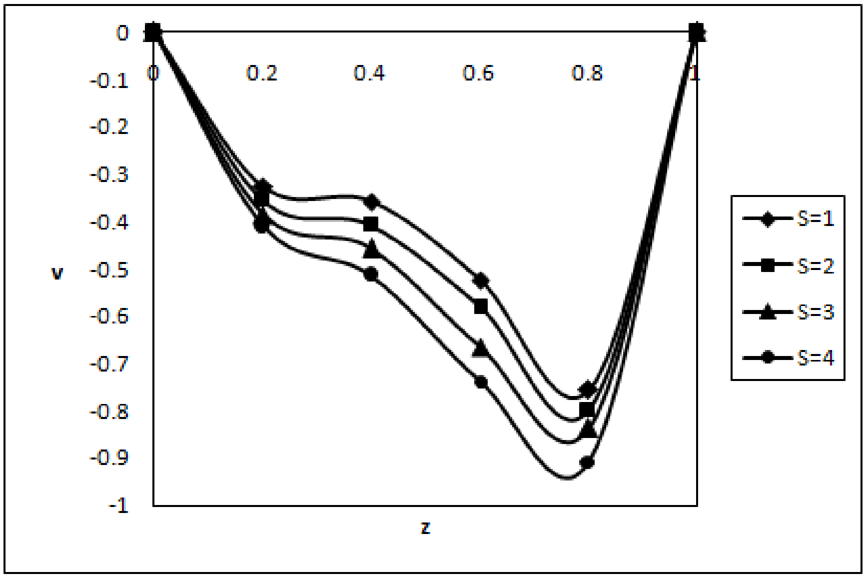

Figure 6: The velocity profile v for different S with $D^{-1}=1000$, $M=2$, $\alpha = \frac{\pi}{4}$

Table 1: The shear stresses (τ_x) on the upper plate.

M	I	II	III	IV	V
2	1.346535	1.321152	1.311534	1.389952	1.397655
5	1.327676	1.307566	1.299952	1.386675	1.353465
8	1.306658	1.287563	1.266523	1.313663	1.327665
10	1.284575	1.266532	1.244523	1.307834	1.315666

	I	II	III	IV	V
D^{-1}	1000	2000	3000	1000	1000
S	1	1	1	2	3

Table 2: The shear stresses (τ_y) on the upper plate.

M	I	II	III	IV	V
2	-0.42756	-0.40057	-0.38465	-0.35753	-0.32116
5	-0.40665	-0.38756	-0.36665	-0.34756	-0.31575
8	-0.38756	-0.36675	-0.34475	-0.31076	-0.28479
10	-0.36657	-0.34466	-0.32115	-0.28475	-0.25766

	I	II	III	IV	V
D^{-1}	1000	2000	3000	1000	1000
S	1	1	1	2	3

Table 3: The shear stresses (τ_x) on the lower plate.

M	I	II	III	IV	V
2	0.665675	0.621573	0.587562	0.704532	0.734523
5	0.607563	0.583246	0.566523	0.657324	0.694531
8	0.585842	0.547563	0.523666	0.635665	0.656759
10	0.566525	0.510518	0.466756	0.596653	0.623563

	I	II	III	IV	V
D^{-1}	1000	2000	3000	1000	1000
S	1	1	1	2	3

Table 4: The shear stresses (τ_y) on the lower plate.

M	I	II	III	IV	V
2	-0.07566	-0.07245	-0.07011	-0.07105	-0.06853
5	-0.07326	-0.06845	-0.06632	-0.07045	-0.06566
8	-0.07157	-0.06375	-0.06007	-0.06895	-0.06224
10	-0.06966	-0.05875	-0.05251	-0.06533	-0.06005

	I	II	III	IV	V
D^{-1}	1000	2000	3000	1000	1000
S	1	1	1	2	3

Table 5: The Discharge (***Q***)

M	I	II	III	IV	V
2	1.401254	1.388985	1.254968	1.568898	1.988596
5	1.300548	1.245448	1.002545	1.425569	1.765897
8	1.211455	1.024554	0.854689	1.336526	1.502456
10	0.998546	0.245877	0.002455	1.114586	1.312544

	I	II	III	IV	V
D^{-1}	1000	2000	3000	1000	1000
S	1	1	1	2	3

Conclusions

1. The magnitude of the velocity component *u* reduces and *v* increases with increasing the intensity of the magnetic field *M,* also the resultant velocity experiences retardation with increasing *M.*
2. Both the velocity components *u* and *v* reduces with increasing the inverse Darcy parameter D^{-1}. Lower the permeability of the porous medium lesser the fluid speed in the entire fluid region. The resultant velocity experiences retardation with increasing the inverse Darcy parameter D^{-1}.
3. We observe that the retardation due to an increase in the porous parameter is more rapid than that due to increase in the Hartmann number *M*. In other words, the resistance offered by the porosity of the medium is much more than the resistance due to the magnetic lines of force.

4. We observed that u exhibits a great enhancement in contrast to v which retards appreciably with increase in the couple stress parameter S, but the resultant velocity shows and appreciable enhancement with in S.
5. The magnitude of these stresses at the upper plate is significantly high compared to the respective magnitudes at the lower plate.
6. The discharge Q reduces in general with increase in the intensity of the magnetic field M and D^{-1} and enhances with the couple stress parameter S.

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Appendix:

$$A = -B - C - D - \frac{P}{M^2 Sin^2\alpha + D^{-1}}$$

$$B = \frac{P(e^{m_1} - 1)}{\left(M^2 Sin^2\alpha + D^{-1}\right)(e^{m_2} - e^{m_1})} - C\frac{(e^{-m_1} - e^{m_1})}{(e^{m_2} - e^{m_1})} - D\,\frac{(e^{-m_1} - e^{m_1})}{(e^{m_2} - e^{m_1})}$$

$$C = \frac{P\,m_1^2(e^{m_2} - 1) - P\,m_2^2(e^{m_1} - 1)}{\left(M^2 Sin^2\alpha + D^{-1}\right)(e^{m_1} - e^{-m_1})} - D\,\frac{(m_2^2 - m_1^2)(e^{m_2} - e^{-m_2})}{(e^{m_1} - e^{-m_1})}$$

$$D = \frac{g_1}{g_2}$$

$$g_1 = [P(m_1^2 - m_2^2)e^{m_1+m_2} + Pm_2^2 e^{m_2} - Pm_1^2 e^{m_1}](m_1^2 - m_2^2)(e^{-m_1} - e^{m_1}) -$$

$$-(m_1^2 - m_2^2)(-e^{m_1+m_2} + e^{-m_1+m_2})(Pm_1^2 e^{m_1} - Pm_2^2 e^{m_2} + Pm_2^2 - Pm_1^2)$$

$$g_2 = \left(M^2 Sin^2\alpha + D^{-1}\right)(e^{m_2} - e^{m_1})[(m_1^2 - m_2^2)e^{-m_1} + (-m_1^2 + m_2^2)e^{m_1}]^2$$

$$[(m_1^2 - m_2^2)e^{m_1-m_2} - (m_1^2 - m_2^2)e^{m_1+m_2}] - [(m_1^2 - m_2^2)e^{-m_1+m_2} - (m_1^2 - m_2^2)e^{m_1+m_2}]$$

$$[(m_1^2 - m_2^2)e^{-m_2} + (-m_1^2 + m_2^2)e^{m_2}]$$

$$m_1 = \sqrt{\frac{1 + \sqrt{1 - 4S\left(M^2 Sin^2\alpha + D^{-1}\right)}}{2S}},$$

$$m_2 = \sqrt{\frac{1 - \sqrt{1 - 4S\left(M^2 Sin^2\alpha + D^{-1}\right)}}{2S}}.$$