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Research Article

EFFECT OF GEOMETRICAL PARAMETERS ON HEAT TRANSFER PERFORMACE OF RECTANGULAR CIRCUMFERENTIAL FINS

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ABSTRACT
Over the years efficiency of a fin has been used as a measure of its performance. However, if heat transfer through fins is given more importance, fin effectiveness is more relevant for this purpose. Variation of these two parameters is of opposite nature with respect to fin geometry parameters. In this work a comparison of these two performance parameters is made for different geometrical proportions of rectangular circumferential fins. Heat transfer rate per unit volume of fin is identified as a new parameter for assessment of fin performance and design. Importance of this parameter is established through study of variation of its variation with fin geometry parameters. The method of analysis presented in this work may be used to ensure design adequacy of fins. The overall conclusion of this study is that designed should air at thin and short fins for best results.

Keywords: *Fins; Rectangular Circumferential Fin; Fin Efficiency; Fin Effectiveness*

Notations

h	Convective heat transfer coefficient at fin surface
k	Thermal conductivity of fin material
m	$\sqrt{\frac{2t}{k\delta_0}}$
q	Heat transfer rate
r	Radius at arbitrary located point on the fin
r_0	Base radius of fin
r_e	Outer radius of fin
t	Temperature at a point located at radius r
t_0	Temperature at root (base radius) of fin
t_s	Ambient temperature
E	Fin effectiveness
I_0, I_1	Bessel's function of first kind and its derivative, respectively
K_0, K_1	Bessel's function of second kind and its derivative, respectively
δ_0	Fin thickness
θ	$t - t_s$
θ_0	$t_0 - t_s$
ρ	r_0/r_e
η	Fin efficiency

INTRODUCTION

Heat transfer through rectangular circumferential fins is determined by taking into account the conduction through the material followed by convective heat transfer to the surroundings. The equations thus obtained yield a solution involving Bessel's functions (Kern, 1972). Boundary layer method has also been used in early research to obtain alternative solutions in polynomial form (Bharani *et al.*, 1992). Both efficiency and effectiveness of a fin have been identified as the guiding parameters for assessment of adequacy of its design (Bharani *et al.*, 1992; Kraus *et al.*, 2000). But the variation of these two parameters is of opposite nature for different parameters defining fin geometry like fin volume, fin length, and fin thickness. Hence, the designer has to compromise over final values of these two parameters. Experimental techniques have also been used in the past to obtain the optimum solution to this problem (Rizzi and Catton, 2003). However, application oriented approach clearly indicates that more emphasis should be given to effectiveness in design of finned surfaces for heat transfer (Bharani *et al.*, 1992; Razelos, 2003; Pawar *et al.*, 2005).

The present work examines efficiency and effectiveness, and also introduces heat transfer rate per unit volume of fin as a new parameter for assessing adequacy of fin design. The method of analysis presented in this work may be used to ensure design adequacy of fins. The overall conclusion of this study is that designed should air at thin and short fins for best results.

Analysis of Rectangular Circumferential Fin

This section presents the mathematical foundation for the analysis of rectangular circumferential fins, followed by sample calculation.

Mathematical Model

Figure 1 shows the rectangular circumferential fin. The heat transfer through such fin is described by the following differential equation[4],

$$r^2 \frac{d^2\theta}{dr^2} + \frac{d\theta}{dr} - m^2 r^2 \theta = 0 \quad (1)$$

The general solution of this equation is,

$$\theta = C_1 I_0(mr) + C_2 K_0(mr) \quad (2)$$

Assuming no heat transfer through the edge of the fin, the solution is,

$$\theta = \frac{\theta_0 \left[K_1(mr_e) I_0(mr) + I_1(mr_e) K_0(mr)\right]}{I_0(mr_e) K_1(mr_e) + I_1(mr_e) K_0(mr_0)} \quad (3)$$

The heat transfer rate is given by,

$$q = 2\pi k r_0 m \theta_0 \left[\frac{I_1(mr_e) K_1(mr_0) - K_1(mr_e) I_1(mr_0)}{I_0(mr_0) K_1(mr_e) + I_0(mr_e) K_0(mr_0)}\right] \quad (4)$$

Effectiveness is given by,

$$E = \frac{km}{h}\left[\frac{K_1(mr_0) I_0(mr_e) - K_1(mr_e) I_1(mr_0)}{I_0(mr_0) K_1(mr_e) + I_1(mr_e) K_0(mr_0)}\right] \quad (5)$$

Efficiency is given by,

$$\eta = \frac{2\pi r_0 \delta_0 k m \theta_0}{2\pi (r_e^2 - r_0^2) h \theta_0 I_0} \times \frac{I_1(mr_e) K_1(mr_0) - K_1(mr_e) I_1(mr_0)}{I_0(mr_0) K_1(mr_e) + I_1(mr_e) K_0(mr_0)} \quad (6)$$

Hence,

$$\eta = \frac{2\rho}{\phi(1+\rho)} \frac{I_1\left[\frac{\phi}{(1-\rho)}\right] K_1\left[\frac{\rho\phi}{(1-\rho)}\right] - K_1\left[\frac{\phi}{(1-\rho)}\right] I_1\left[\frac{\rho\phi}{(1-\rho)}\right]}{I_0\left[\frac{\rho\phi}{(1-\rho)}\right] K_1\left[\frac{\phi}{(1-\rho)}\right] + I_1\left[\frac{\phi}{(1-\rho)}\right] K_0\left[\frac{\rho\phi}{(1-\rho)}\right]} \quad (7)$$

Where,

$$\phi = r_e(1-\rho)\left(\frac{2h}{k\delta_0}\right)^{1/2} \tag{8}$$

Sample Calculations

The following parameter values are used for the sample calculations presented in this section,
h=20 W/m^{2}K, k=40 W/mK, r_i = 0.025m, δ_0 = 0.002 m, volume of fin 16 cc.

Outer radius of fin, $r_0^2 = \frac{V}{\pi\delta_0} + r_i^2 \Rightarrow = r_0 = 0.056$ m

Heat dissipation area of fin = $\frac{V}{\delta_0}$ = 0.008 m^2

Characteristic length $L = \frac{r}{\delta_0} \Rightarrow L_i = 12.5, L_o = 28$

Biot number $B_i = \frac{h\delta_0}{2k}$ = 0.0005

Also, $m = \sqrt{\frac{2h}{k\delta_0}}$ =22.3606

The radius ratio is, = 0.4464

And, $\phi = r_0(1-\rho)\sqrt{\frac{2h}{k\delta_0}}$ = 0.6932

Now heat transferred can be calculated by,

$$q = 2\pi k r_0 m\theta_0\left[\frac{I_1(mr_e)K_1(mr_0) - K_1(mr_e)I_1(mr_0)}{I_0(mr_0)K_1(mr_e) + I_0(mr_e)K_0(mr_0)}\right]$$

For , θ_o = 100^0, q = 25.819 W

Then efficiency is,

$$\eta = \frac{2\rho}{\phi(1+\rho)}\frac{I_1\left[\frac{\phi}{(1-\rho)}\right]K_1\left[\frac{\rho\phi}{(1-\rho)}\right] - K_1\left[\frac{\phi}{(1-\rho)}\right]I_1\left[\frac{\rho\phi}{(1-\rho)}\right]}{I_0\left[\frac{\rho\phi}{(1-\rho)}\right]K_1\left[\frac{\phi}{(1-\rho)}\right] + I_1\left[\frac{\phi}{(1-\rho)}\right]K_0\left[\frac{\rho\phi}{(1-\rho)}\right]} = 0.80685$$

Effectiveness is,

$$E = \frac{km}{h}\left[\frac{K_1(mr_0)I_0(mr_e) - K_1(mr_e)I_1(mr_0)}{I_0(mr_0)K_1(mr_e) + I_1(mr_e)K_0(mr_0)}\right] = 41.114$$

RESULTS AND DISCUSSION

In the present study the fin thickness was varied from 2 mm to 10 mm in intervals of 2 mm. The performance parameters used in the study are fin efficiency and fin effectiveness. The material volume range was varied from 18 cc to 80 cc in equal steps.

Effect of Outer Radius of the Fin on Performance Parameters

The variation in performance parameters for constant fin thickness and base radius is shown in Table 1. Figure 2 shows the variation of performance parameters with respect to the outer radius of fin, for a constant fin thickness of 2 mm and a constant base radius of 25 mm. The efficiency decreases from 80%

to 30% while the effectiveness increases from 40 to 74. Figure 3 shows variations for fin thickness of 4 mm with base radius of 25 mm. Here the efficiency initially increases from 80% to 90% and then decreases to 67%, while the effectiveness increases steadily from 39 to 74. Figure 4 shows the variations for fin thickness of 6 mm and base radius of 25 mm.
Here the decrease in efficiency is from 99% to 85% accompanied by an increase in the effectiveness from 7 to 25.

Figure 5 corresponds to fin thickness of 8 mm and base radius of 25 mm. The efficiency is seen to decrease from 99% to 92% and effectiveness is seen to increase from 4 to 15.

Figure 6 shows variations for fin thickness of 10 mm and base radius of 25 mm. The efficiency in this case decreases from 99.5% to 96% while the effectiveness increases from 3 to 10.

These results clearly show that the efficiency and the effectiveness vary in opposite manner when the outer radius of the fin (and hence the material volume) is increased. It can be noted that the heat transfer rate increases continuously with increase in outer radius (and hence the material volume), for constant fin thickness and constant base radius. Thus, in order to get maximum heat transfer rate, the outer radius should be maximum.

However, as indicated by the variation of the effectiveness, both the curves of efficiency and effectiveness become flatter with increasing value of outer radius or the material volume.

Thus the relative advantage of larger outer radius of fin (and larger material volume) diminishes continuously with increase in outer radius. Also, as evident from the graph, the efficiency shows a continuous decrease with increasing outer radius.

Since the basic purpose of a fin is to maximize the heat transfer from the parent body, it is obvious that a larger value of effectiveness is more relevant in assessment of fin performance. It is also quite obvious from these graphs that the effectiveness is high for higher outer radii (larger material volume) of the fin, for which the efficiency is lower.

Similarly, for lower values of outer fin radii (smaller material volume), the efficiency is higher but the effectiveness is low.

Effect of Fin Volume on Performance Parameters

The ability of heat transfer through fin is reflected by its effectiveness. Further, the cost of manufacturing, among other factors, mainly depends upon the quantity of the material.

Therefore design adequacy of a fin geometry, both in terms of heat transfer rate and cost effectiveness, can be ascertained through study of variation in performance parameters with variation in fin thickness, keeping the material volume constant.

The performance parameters were determined for fin thickness range of 2 mm to 10 mm, for constant base radius of 25 mm, and are presented in Table 2.

Figure 7 shows variation of performance parameters for constant material volume of 16 cc, Figure 8 for 32 cc, Figure 9 for 48 cc, Figure 10 for 64 cc and Figure 11 for 80 cc.

Here again it is observed that effectiveness and efficiency vary in opposing manner as the fin thickness is increased.
Also, it is observed that a larger fin thickness gives higher efficiency but lower effectiveness. Keeping in mind that these variations are for a constant volume, and therefore a constant material and manufacturing cost in general, it is obvious that best effectiveness can be obtained by keeping the fin thickness at the minimum.
The practical limits on minimum fin thickness will be imposed by limitations of the manufacturing process, which is mostly pressure die-casting these days.

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Table 1: Geometrical & Performance Parameters (Base Radius 25 mm, Biot No. $5x10^4$)

Fin Thickness, mm	Outer Radius, mm	Area, mm^2	Volume, cc	Heat Transfer Rate, W/m^2	Effectiveness	Efficiency
2	0.056	80	16	25.82	41.11	80.69
	0.076	160	32	37.78	60.16	77.39
	0.091	240	48	42.93	68.36	44.72
	0.104	320	64	45.34	72.20	35.42
	0.116	400	80	46.55	74.13	29.10
4	0.044	40	16	15.42	12.27	80.69
	0.056	80	32	28.54	22.72	89.17
	0.067	120	48	38.97	31.04	81.22
	0.076	160	64	47.10	37.50	73.59
	0.084	200	80	53.35	42.48	66.69
6	0.038	26.67	16	10.54	5.59	98.78
	0.048	53.33	32	20.48	10.87	96.02
	0.056	80.00	48	29.59	15.71	92.48
	0.063	106.67	64	37.79	20.06	88.56
	0.070	133.33	80	45.08	23.93	84.52
8	0.036	20	16	7.96	3.17	99.45
	0.044	40	32	15.70	6.25	98.14
	0.050	60	48	23.12	9.20	96.34
	0.056	80	64	30.16	12.01	94.24
	0.062	100	80	36.77	14.64	91.94
10	0.034	16	16	6.38	2.03	99.71
	0.041	32	32	12.67	4.04	98.98
	0.046	48	48	18.81	6.00	97.96
	0.052	64	64	24.76	7.89	96.72
	0.056	80	80	30.51	9.72	95.33

Table 2: Performance Parameters (Base Radius 25 mm)

Fin Volume, cc	Fin Thickness, mm	Outer Radius, mm	Heat Transfer Rate	Effectiveness	Efficiency %
16	2	56	25.81	41.11	80.68
	4	44	15.41	12.27	96.35
	6	38	10.53	5.59	98.78
	8	36	7.95	3.16	99.45
	10	34	6.381	2.032	99.709
32	2	76	37.78	60.16	59.03
	4	56	28.53	22.72	89.17
	6	48	20.48	10.87	96.02
	8	44	15.70	6.25	98.14
	10	41	12.67	4.04	98.98
48	2	91	42.93	68.36	44.72
	4	67	38.99	31.04	81.22
	6	56	29.59	15.71	92.48
	8	50	23.12	9.20	96.34
	10	46	18.81	5.99	97.96
64	2	104	45.34	72.19	35.42
	4	76	47.10	37.50	73.59
	6	63	37.79	20.06	88.56
	8	56	30.16	12.01	94.24
	10	52	24.76	7.87	96.72
80	2	116	46.55	74.13	29.10
	4	84	53.35	42.48	66.69
	6	70	45.08	23.93	84.52
	8	62	36.77	14.64	91.94
	10	56	30.51	9.72	95.33

Table 3: Fin Effectiveness for Different Fin Volumes and Thicknesses (Base Radius 25 mm)

Fin Thickness, mm	Volume 16 cc	Volume 32 cc	Volume 48 cc	Volume 64 cc	Volume 80 cc
2	41.11	60.16	68.36	72.19	74.13
4	12.27	22.18	31.04	37.50	42.48
6	5.59	10.87	15.71	20.06	23.93
8	3.17	6.25	9.20	12.01	14.64
10	2.03	4.04	5.99	7.89	9.72

Table 4: Heat Transfer Rate per unit Volume for Different Fin Volumes and Thicknesses (Base Radius 25 mm)

Fin Thickness mm	Volume 16 cc	Volume 32 cc	Volume 48 cc	Volume 64 cc	Volume 80 cc
2	1.61	1.18	0.89	0.71	0.58
4	0.96	0.89	0.81	0.74	0.67
6	0.66	0.64	0.62	0.59	0.56
8	0.50	0.49	0.48	0.47	0.46
10	0.40	0.40	0.39	0.39	0.38

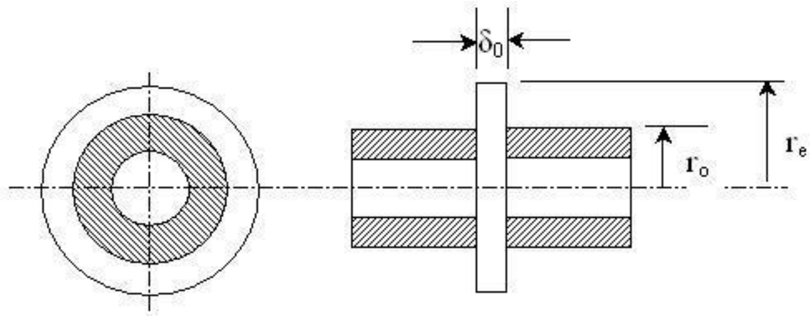

Figure 1: Rectangular Circumferential Fin

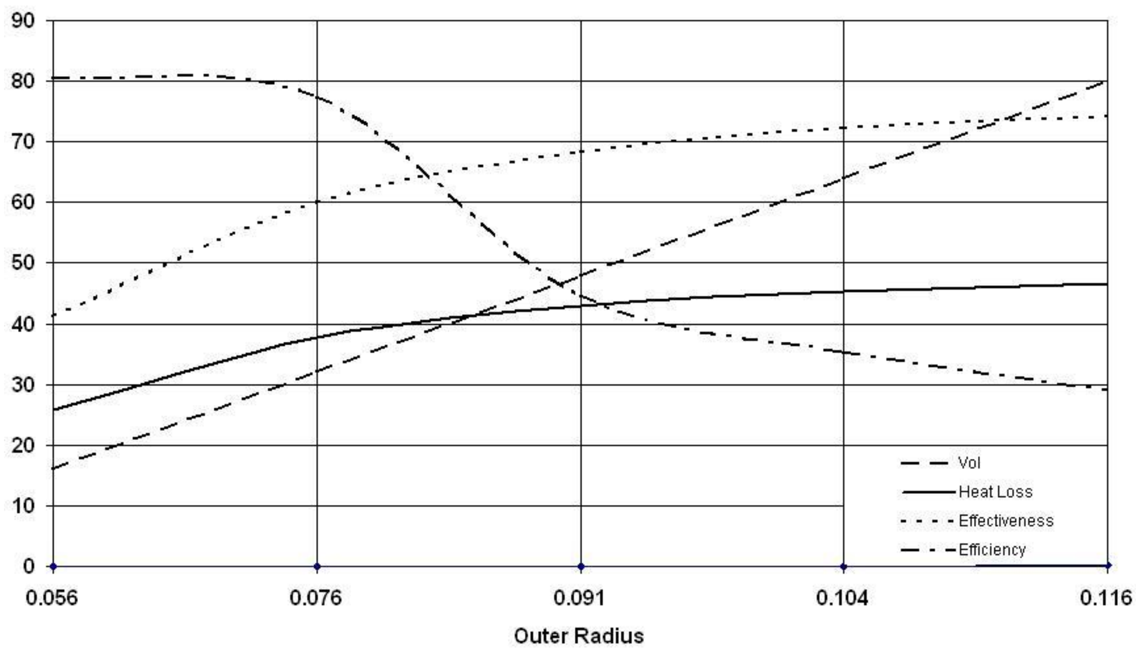

Figure 2: Variation of performance parameters for fin thickness 2 mm

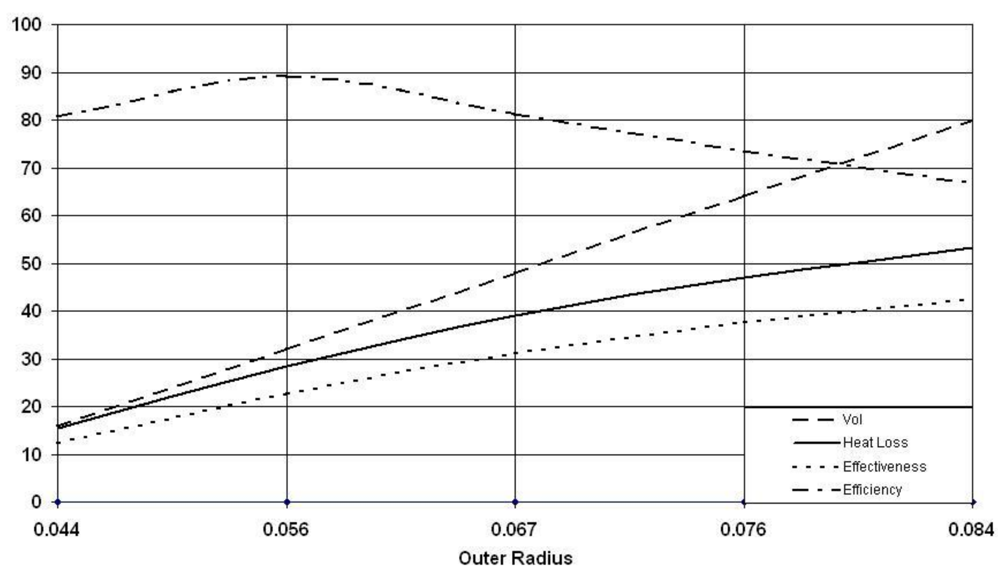

Figure 3: Variation of performance parameters for fin thickness 4 mm

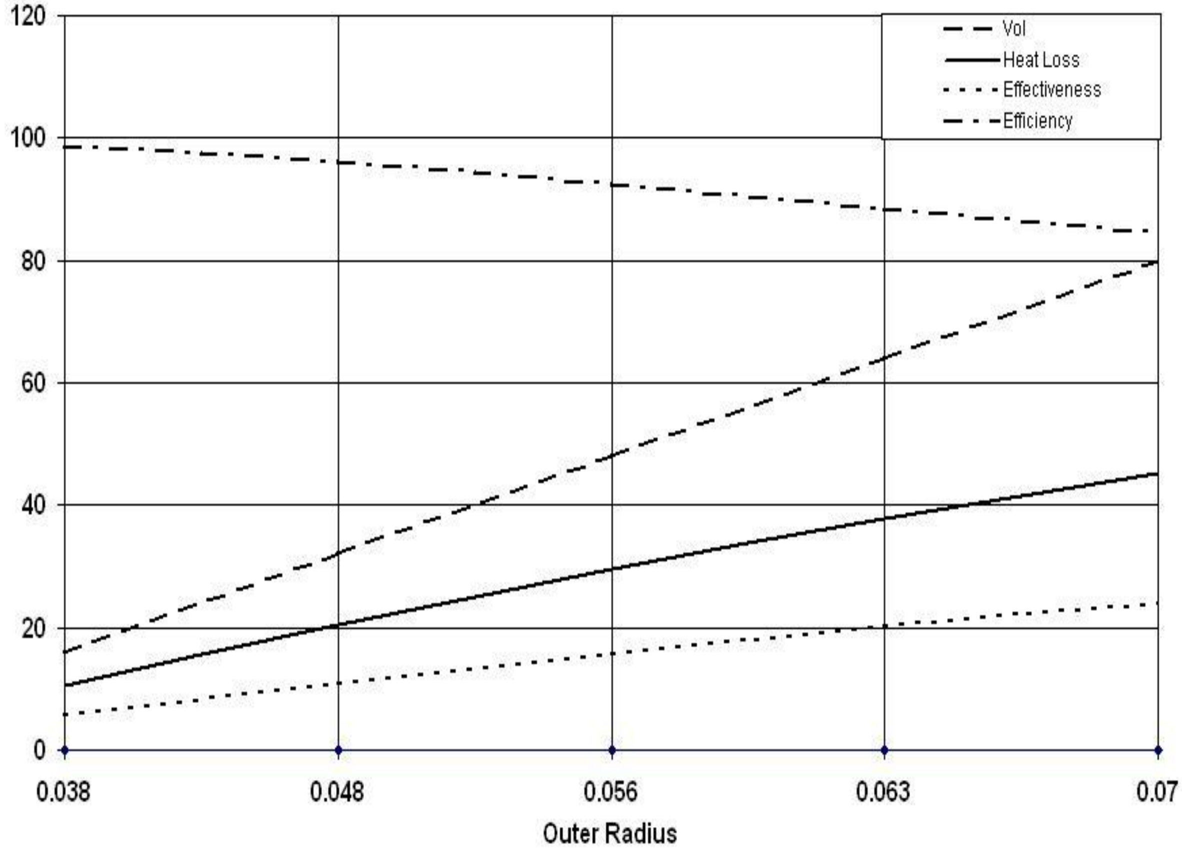

Figure 4: Variation of performance parameters for fin thickness 6 mm

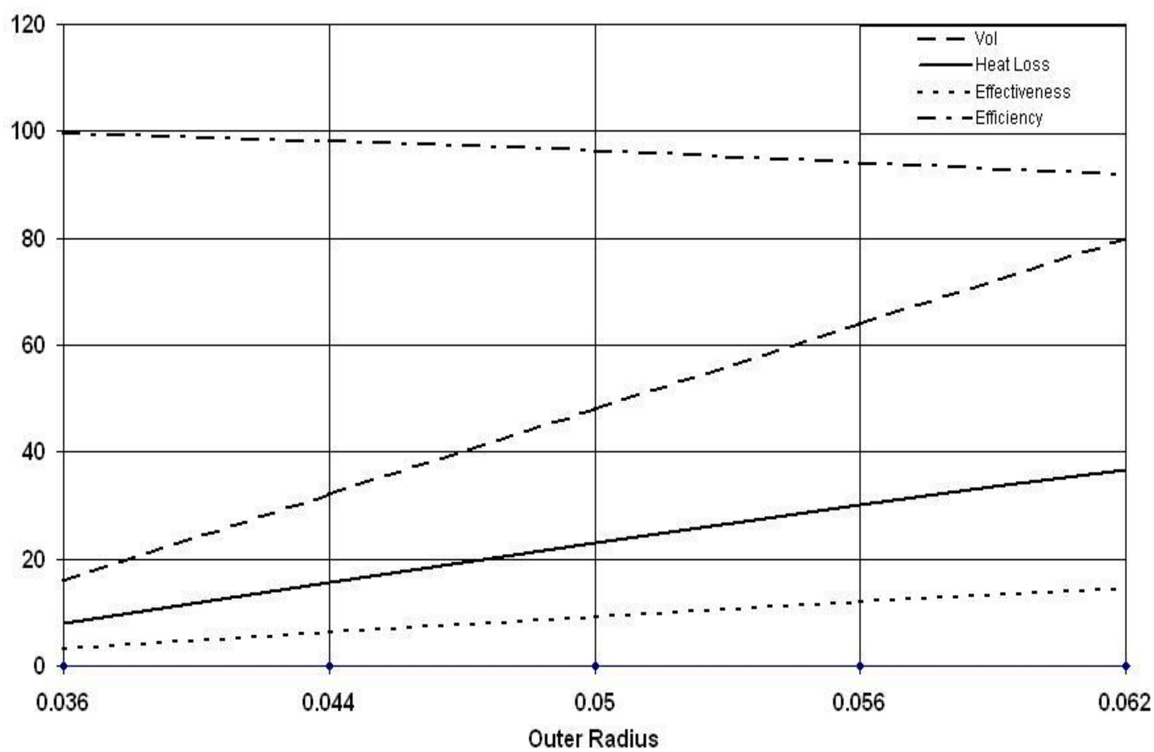

Figure 5: Variation of performance parameters for fin thickness 8 mm

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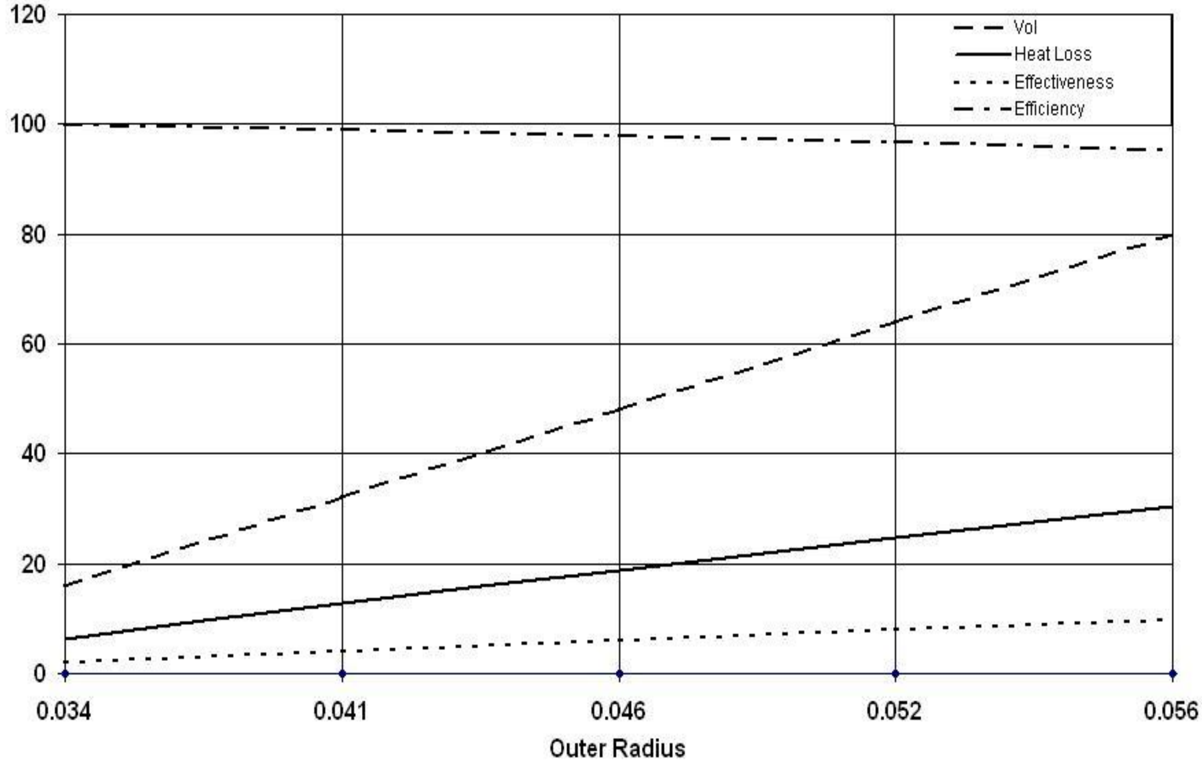

Figure 6: Variation of performance parameters for fin thickness 10 mm

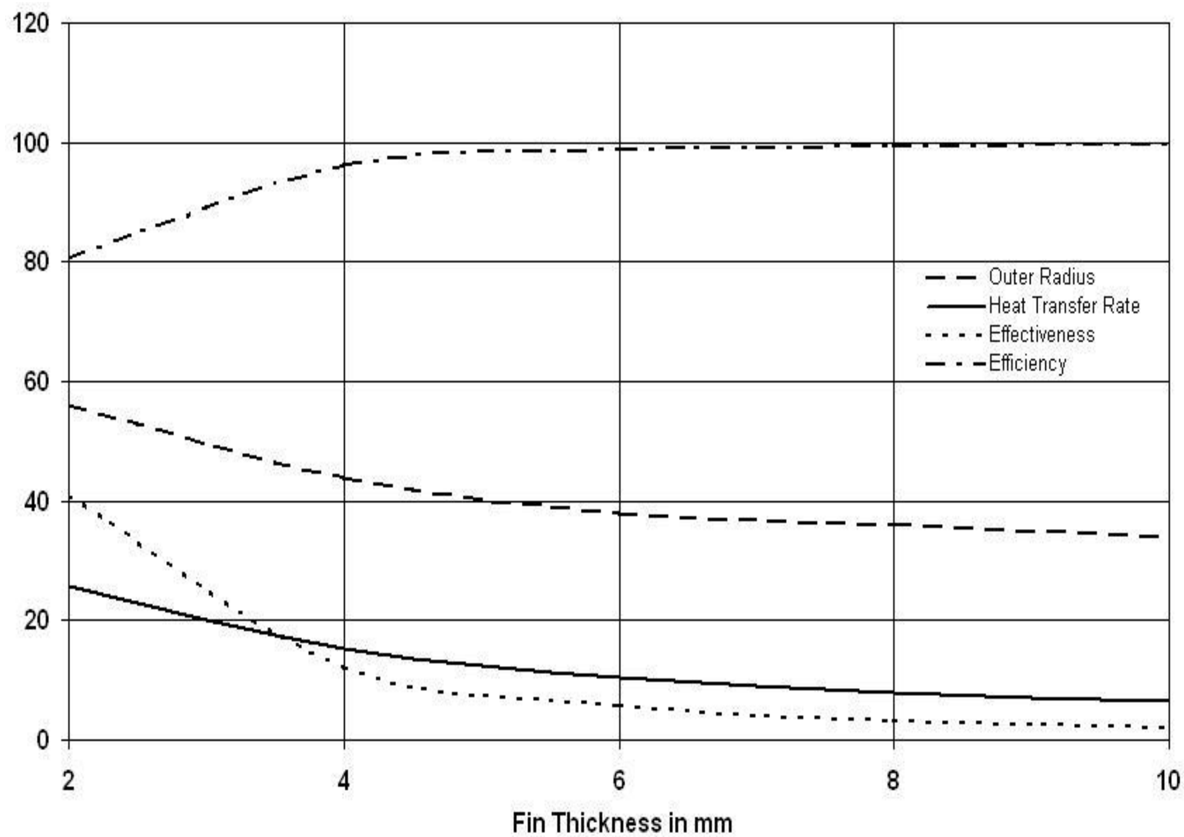

Figure 7: Variation of performance parameters for material volume 16 cc

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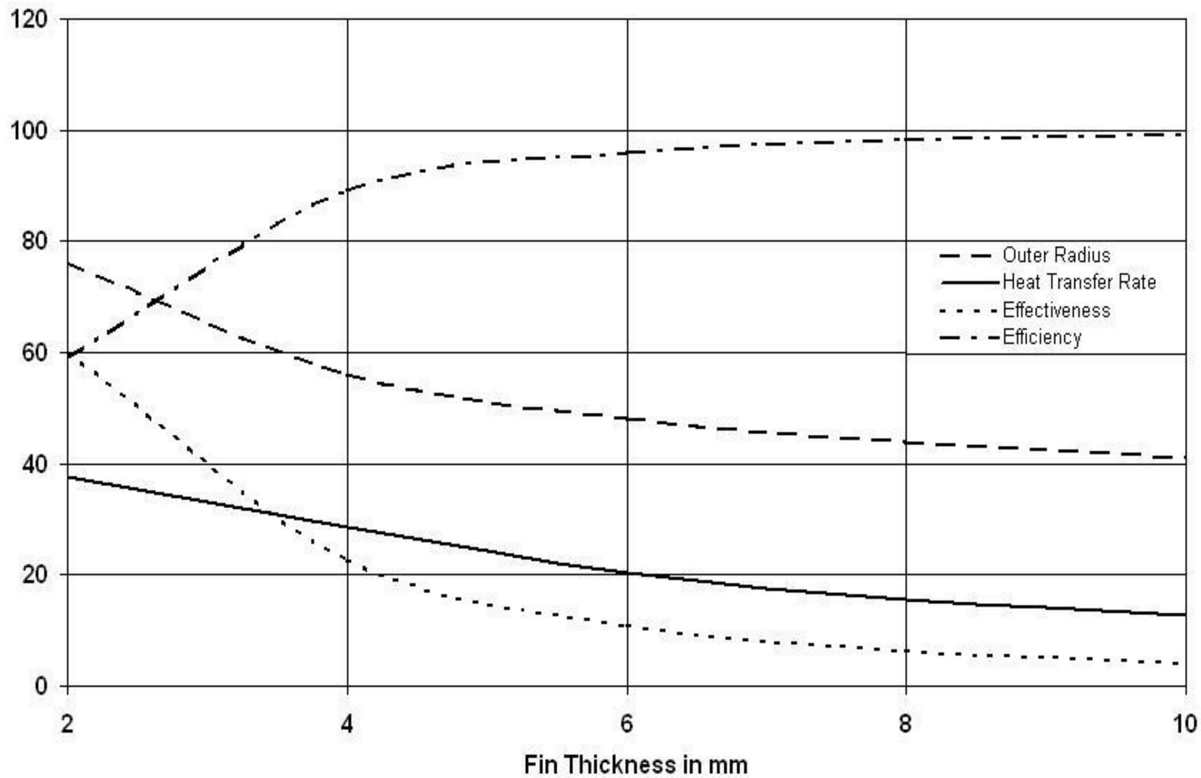

Figure 8: Variation of performance parameters for material volume 32 cc

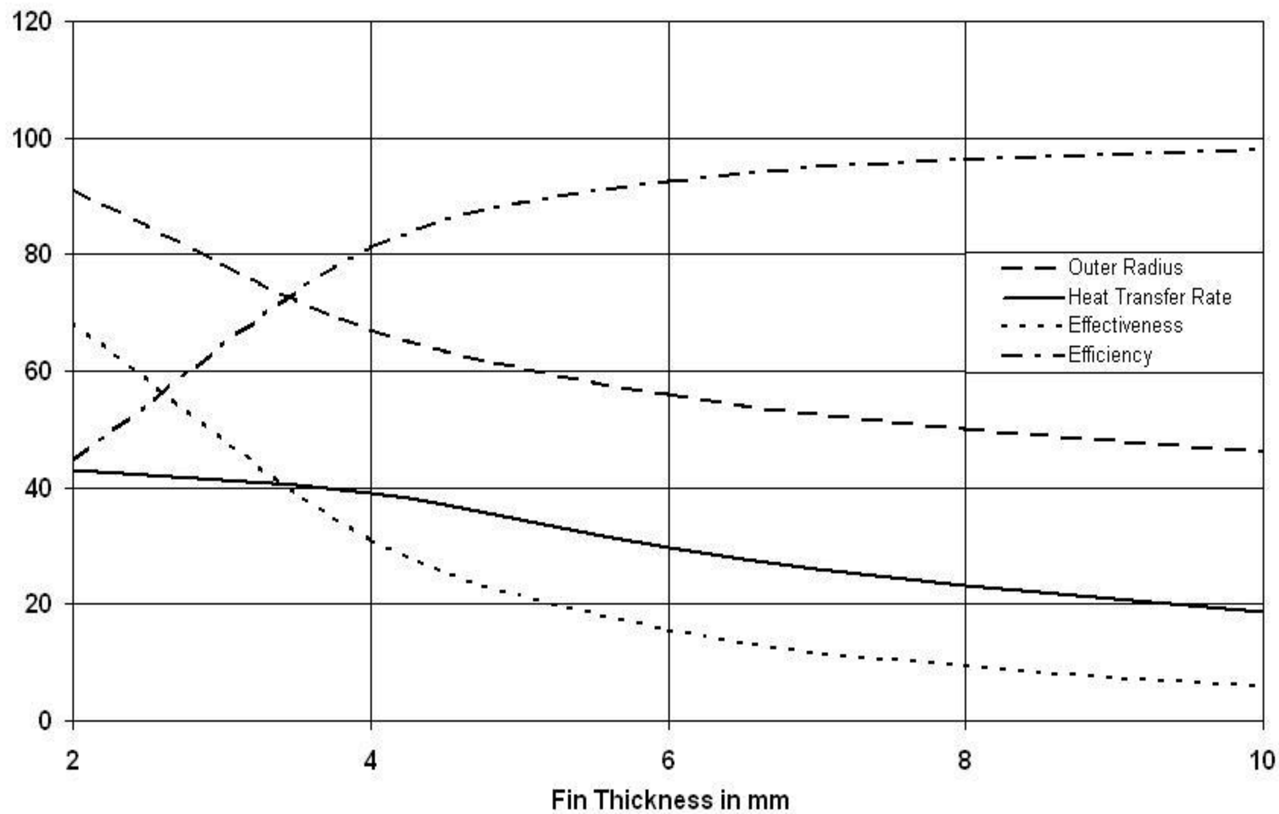

Figure 9: Variation of performance parameters for material volume 48 cc

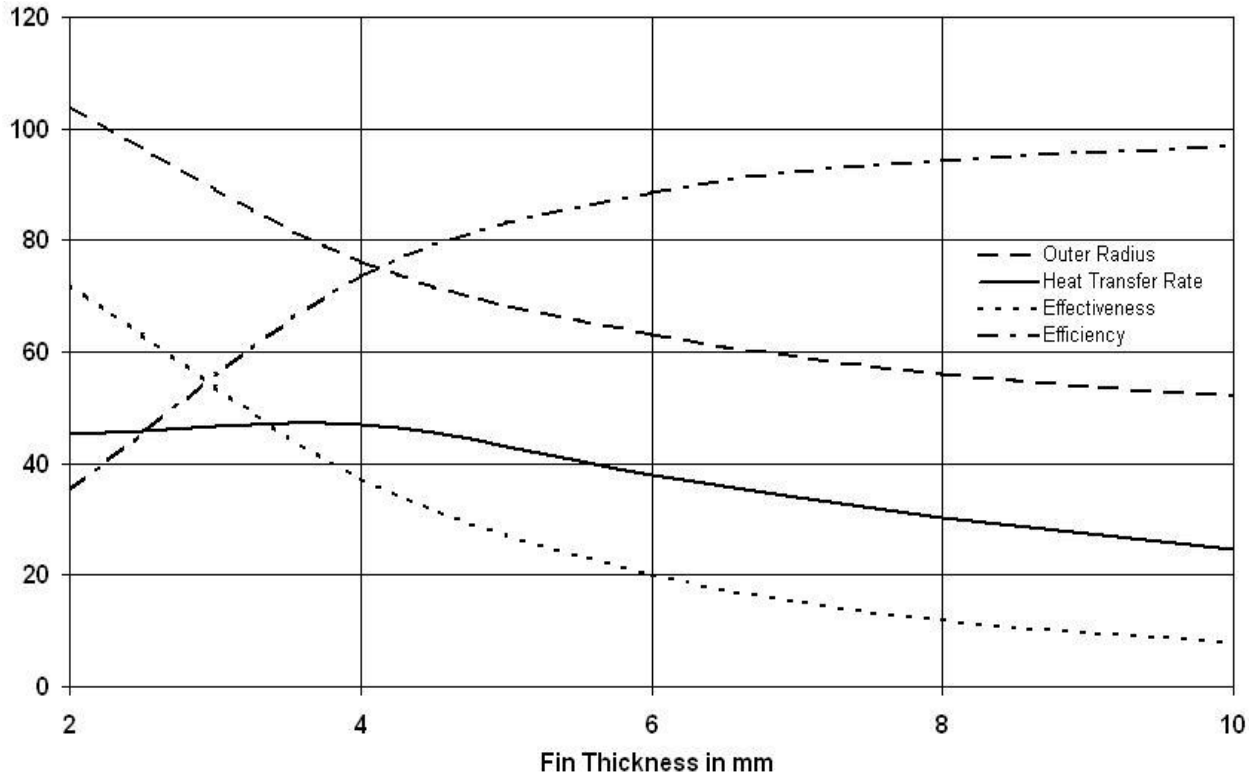

Figure 10: Variation of performance parameters for material volume 64 cc

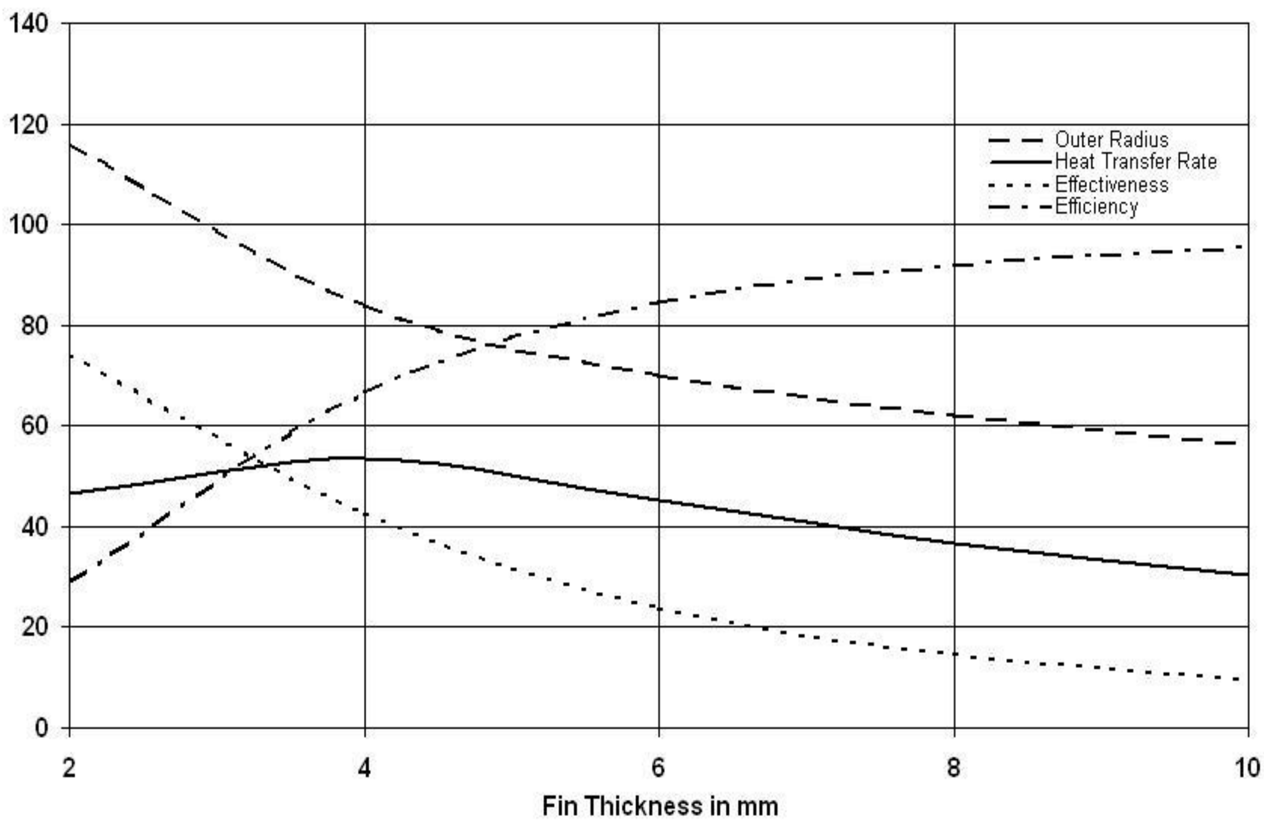

Figure 11: Variation of performance parameters for material volume 80 cc

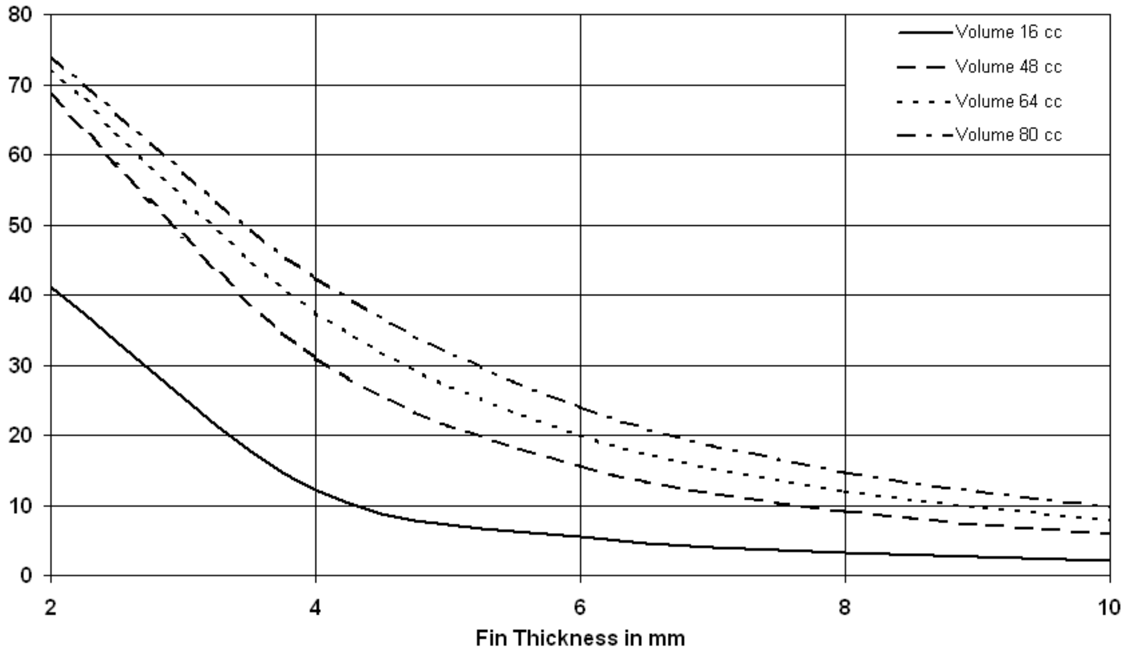

Figure 12: Variation of Effectiveness of constant volume fins for different fin thickness

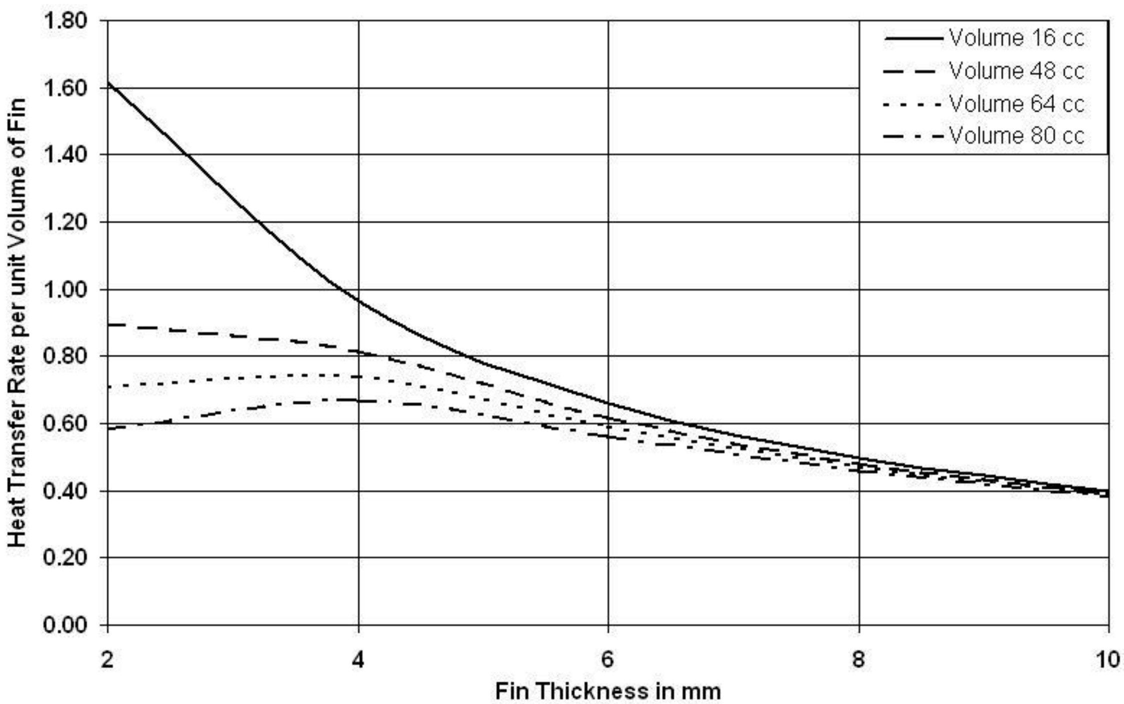

Figure 13: Heat Transfer per unit Volume of Fin

Variation in Effectiveness with Fin Thickness and Fin Volume

In this section effectiveness is examined from the point of view of geometrical parameters involved in fin design. The base radius of fin is generally pre-specified in the form of outer diameter of pipe. Therefore fin geometry can be defined in terms of fin thickness and outer radius. Also, effect of material quantity on performance of fin could be examined more clearly if volume of fin is used in place of outer radius. Table 3 presents the data from the study and Figure 12 shows variation of effectiveness of constant volume fins for different fin thickness. It is clear that smaller fin thickness leads to higher effectiveness, as indicated in the previous sections. It may be observed from Table 3 that for fin thickness of 2 mm, an increase in volume from 16 cc to 32 cc (an increase of 16 cc) results in increase in effectiveness from bout 41.11 to 60.16 (an increase by about 19). But, increasing the volume from 32 cc to 48 cc (again an increase of 16 cc) results in increase in effectiveness from 60.16 to 68.36 (an increase by about 8). Thus, it is observed that increase in material quantity results in less than proportionate improvement in effectiveness. Figure 12 indicates that effectiveness deteriorates rapidly with increase in fin thickness. Therefore, the designer must give due consideration to this fact and consider alternative manufacturing processes in order to achieve low fin thickness. The family of curves shown in Figure 12 may be used by the designer in identifying suitable fin thickness, based upon considerations of cost of manufacturing and available manufacturing processes. Then depending upon the desirable hear transfer rate, the volume can be determined based upon effectiveness.

Variation in Heat Transfer Rate per unit Volume with Fin Thickness and Fin Volume

Heat transfer rate per unit volume of fin should be considered an important parameter for assessment of fin performance. This parameter may be used as an indicator of overall weight of the system. Table 4 shows the values for this parameter for different combinations of fin thicknesses and fin volumes. These results are plotted in Figure 13. It is observed that this parameter is very sensitive to both fin thickness and fin volume. Also, unlike the other performance parameters like fin effectiveness and fin efficiency its behavior is not monotonous. Figure 13 shows that maximum heat transfer rate per unit volume of fin may be achieved by selecting a combination of small fin thickness and small fin volume. A small fin volume, for a given fin thickness and base radius, would lead to a small outer radius of fin. Thus, in order to achieve a high heat transfer rate per unit volume, the designer should aim at small fin thickness combined with small fin height. This may lead to increasing the number of fins to achieve desired overall heat transfer rate, but would ensure optimum design from the point of view of material utilization as well as weight of the system.

The curve corresponding to fin volume of 80 cc clearly indicates that excessively large volume may lead to reduced heat transfer per unit volume of fin, if the fin thickness is not selected judiciously. In this manner, analysis of this parameter is useful in avoiding such pit-falls in design of finned surfaces.

CONCLUSION

Analysis of rectangular circumferential fins was performed in order to study the effect of fin geometry on fin performance parameters. The following are the main conclusions based on this work:

1. Variations of effectiveness and efficiency are of opposite nature for different parameters of fin geometry. Fin effectiveness should be given priority in fin design from the point of view of heat transfer rate through the fin. Therefore the objective of fin design should be to maximize fin effectiveness.
2. For constant fin thickness and base radius higher fin effectiveness may be achieved through larger outer radius of fin, although the value of efficiency reduces substantially in this case.
3. It was also observed that for constant fin volume (material quantity) and constant base radius higher fin effectiveness may be achieved by keeping the value of fin thickness as low as practicable. Further, the effect of fin thickness on effectiveness was found to be more prominent compared to fin volume.
4. Heat transfer rate per unit fin volume was identified as a critical parameter in fin design, leading to optimum material utilization and overall weight of the system. Study of this parameter indicates that the designer should aim for fins of small thickness as well as small height. Hence the logical approach for fin design should be to select a small fin thickness, based on manufacturing and cost considerations, followed by determination of the fin volume to achieve desired fin effectiveness and heat transfer rate. This approach should result in optimum fin geometry both from heat transfer and cost point of view.

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