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Research Article

AN INTERESTING PROBLEM RELATED TO PROGRESSION IN ALGEBRA

***Gopalan M.A., Vidhyalakshmi S. and Presenna R.**
Department of Mathematics, SIGC, Trichy-620002, Tamilnadu
**Author for Correspondence*

ABSTRACT
Given the product value of two non-zero distinct integers x, y and the value of the ratio $\frac{\text{Arithmetic mean of gcd and lcm of } x, y}{\text{Harmonic mean of gcd and lcm of } x, y}$, a process of obtaining the values of x and y is illustrated.

Keywords: *Arithmetic mean, Harmonic Mean*
MSC classification number: 11D99

INTRODUCTION
Every mathematics student is familiar with the concept of Arithmetic mean, Geometric mean and Harmonic mean between any two Natural numbers. For a variety of problems on Arithmetic mean, Geometric mean and Harmonic mean, one may refer (Bernald and Child, 2006; Hall and Knight, 1998). In page 91 of the journal "The Mathematics Teacher" published by the association of the Mathematics teacher of India, there is a problem involving Arithmetic mean and Harmonic mean. In this short communication an attempt has been made to generalize the above problem.

MATERIALS AND METHOD
Method of Analysis
Let x, y be any two non-zero distinct positive integers whose product is denoted by P.
Let p= gcd (x,y),q= lcm (x,y). Note that q>p and xy=pq=P. Let A be an arithmetic mean of p,q and H be a Harmonic mean of p,q. Denote the value of the ratio $\frac{A}{H}$ by N_1, a rational number.

Given the values of P and N_1, we illustrate below a process of obtaining the values of x and y. Consider

$$\frac{A}{H} = N_1 \Rightarrow (q+p)^2 = 4PN_1$$

$$q + p = 2\sqrt{PN_1} \tag{1}$$

We have the identity

$$(q-p)^2 = (q+p)^2 - 4pq$$

$$\Rightarrow (q-p)^2 = 4PN_1 - 4P$$

$$(q-p) = 2\sqrt{P(N_1-1)} \tag{2}$$

Sovlving (1) and (2), we get

$$q = \sqrt{PN_1} + \sqrt{P(N_1-1)}$$

$$p = \sqrt{PN_1} - \sqrt{P(N_1-1)}$$

It is note worthy that the products PN_1 and $P(N_1-1)$ should be perfect squares. Since p is the gcd (x,y), we write

x=pX, y=pY, where gcd (X,Y)=1 (3)

Now, $P = xy = p^2 XY$

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$$XY = \frac{P}{p^2} \tag{4}$$

Which should be an integer. Then, it is possible to choose X,Y such that gcd (X,Y)=1. Substituting the values of p, X, Y in (3), the values of x, y are obtained.
A few examples are given below.

S.no	P	N_1	PN_1	$P(N_1-1)$	p	XY	X	Y	x	y
1	192	$\frac{169}{48}$	676	484	4	12	4 12	3 1	16 48	12 4
2	192	$\frac{4}{3}$	256	64	8	3	3	1	24	8
3	60	$\frac{64}{15}$	256	196	2	15	5	3	10	6
4	36	$\frac{25}{9}$	100	64	2	9	9	1	18	2
5	24	$\frac{98}{48}$	49	25	2	6	3	2	6	4
6	256	$\frac{100}{64}$	400	144	8	4	1	4	8	32
7	100	$\frac{169}{25}$	676	576	2	25	1	25	2	50

CONCLUSION

To sum up, the following three conditions have to be satisfied

1) PN_1 is a perfect square
2) $P(N_1-1)$ is also a perfect square
3) $P \equiv 0 (\text{mod } p^2)$

Also from the above table, it is observed that when P is a perfect square the ratio $\frac{(\text{Geometric mean of p,q})^2}{\text{Harmonic mean of p,q}}$ is an integer representing hypotenuse of a Pythagorean triangle.

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