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Research Article

DIOPHANTINE QUADRUPLE INVOLVING JACOBSTHAL LUCAS NUMBER AND THABIT-IBN-KURRAH NUMBER WITH PROPERTY $D(1)$

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ABSTRACT
We search for three distinct integers a,b,c such that product of any two from the set added with 1 is a perfect square. Also, we show that the triple can be extended to the quadruple with property $D(1)$

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NOTATIONS

TK_n : Thabit-ibn-kurrah number of rank n

j_n : Jacobsthal Lucas number of rank n

J_n : Jacobsthal number of rank n

Ky_n : Kynea number of rank n

INTRODUTION

Let n be an integer. A set of m positive integers $(a_1, a_2, a_3, ...a_m)$ is said to have the property $D(n)$,if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ .Such a set is called a diophantine m-tuple or P_n set of size m.The problem of construction of such set was studied by Diophantus

Many mathematicians considered the construction of different formulations of Diophantine triples, quadruple and quintuples with property $D(n)$ for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may refer (Brown, 1985; Bashmakova, 1974; Bugeaud *et al.*, 2007; Dujella, 1997; Filipin, 2009; Filipin and Fujita, 2013; Fujita, 2010; Gopalan and Srividhya, 2012; Gopalan and Srividhya, 2012; Gopalan and Pandichelvi, 2013; Gopalan *et al.*, 2014; Gopalan *et al.*, 2014; Gopalan *et al.*, 2014; Jukic, 2012; Meena *et al.*, 2014; Rich, 2012; Vidhyalakshmi *et al.*, 2014; Andrej) for an extensive review of various problems on Diophantine triples, quadruple and quintuples. This paper aims that at constructing Diophantine quadruples is constructed where the product of any two members of the quadruple with the addition of 1 satisfies the required property.

METHOD OF ANALYSIS

Let $a = 3j_{2n} - 4$, $b = TK_{2n}$ be any two integers such that $ab + 1$ is a perfect square.

Let $C_s(n)$ be any non-zero integer such that

$$(3j_{2n} - 4)C_s(n) + 1 = \alpha_s^2 \tag{1}$$

$$(TK_{2n} + 2)C_s(n) + 1 = \beta_s^2 \tag{2}$$

Eliminating $C_s(n)$ between (1) and (2), we get

$$(TK_{2n}+2)\alpha_s^2-(3j_{2n}-4)\beta_s^2=2 \tag{3}$$

Introducing the linear transformations

$$\left.\begin{aligned}\alpha_s&=X_s+(3j_{2n}-4)T_s\\ \beta_s&=X_s+(TK_{2n}+2)T_s\end{aligned}\right\} \tag{4}$$

in (3),we get

$$X_s^2=(3j_{2n}-4)(TK_{2n}+2)T_s^2+1 \tag{5}$$

This is a well known pellian equation,whose general solution is given by

$$\left.\begin{aligned}X_s&=\frac{1}{2}\left\{\left[TK_{2n}+1+\sqrt{D}\right]^{s+1}+\left[TK_{2n}+1-\sqrt{D}\right]^{s+1}\right\}\\ T_s&=\frac{1}{2\sqrt{D}}\left\{\left[TK_{2n}+1+\sqrt{D}\right]^{s+1}-\left[TK_{2n}+1-\sqrt{D}\right]^{s+1}\right\}\end{aligned}\right\} \tag{6}$$

where $s=-1,0,1,2.......$

$D=(3j_{2n}-4)(TK_{2n}+2)$

Taking $s=0$ in (6) and using (1),we get

$C_0(n)=12j_{2n}-12$

Note that $(a,b,C_0(n))$ is the Diophantine triple with property $D(1)$

Now, substituting $s=1$ in (6) we have

$$\left.\begin{aligned}X_1&=2(TK_{2n}{}^2)+4TK_{2n}+1\\ T_1&=2TK_{2n}+2\end{aligned}\right\} \tag{7}$$

and using (1),we see that

$C_1(n)=16(TK_{2n}+1)^3-12j_{2n}+12$

Thus, we obtain

$(3j_{2n}-4, TK_{2n}+2, 12j_{2n}-12, 16(TK_{2n}+1)^3-12j_{2n}+12)$ as a diophantine quadruple with the property $D(1)$

Some numerical examples are presented below:

Diophantine quadruple with property D(1)

n	$(a,b,C_0(n),C_1(n))$
1	(11,13,48,27600)
2	(47,49,192,1769280)
3	191,193,768,113245440)

The repetition of the above process leads to the generation of many diophantine quadruples with the property D(1)

A Few Interesting Properties Observed are As Follows

1. $C_0(n) - 4b(n) \equiv 0(\bmod 4)$
2. $b(n) - a(n) = 2$
3. $C_0(n)b(n) - 4a(n) \equiv 0(\bmod 4)$
4. $a(n) + b(n) + 6Ky_{2n} - C_0(n) \equiv 0(\bmod 2)$
5. Each of the following is a nasty number

(a) $18[C_1(n) + C_0(n)]$

(b) $3[a(n) + b(n) + C_0(n) - 54J_{2n}]$

6. $4[C_1(n) + C_0(n)]$is a cubic integer
7. $a(n) - b(n) - C_1(n) + \frac{3}{12}(C_0(n))^3 - 36J_{2n} \equiv 0(\bmod 10)$

CONCLUSION

In the construction of the Diophantine quadruples, we have assumed the product ab added with 1 is a perfect square. One may search for Diophantine quadruples consisting of special numbers with property D(n),D(n^2 **)**and so on.

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