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Research Article

ONE INTERESTING FAMILY OF 3-TUPLE

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ABSTRACT
This paper concerns with the study of constructing a special family of 3-tuples (a,b,c) such that the product of any two elements of the set added with their sum is a Perfect square.

Keywords: *Diophantine triple*

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INTRODUCTION

The Problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus (Bashmakova, 1974). A set of positive integers $\{a_1, a_2......a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$, if $a_i a_j + n$, a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the Construction of different formulations of Diophantine triples with property D(n) for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may refer (Thamotherampillai, 1980; Brown, 1985; Gupta and Singh, 1985; Beardon and Deshpande, 2002; Deshpande, 2002; Deshpande, 2003; Bugeaud *et al.*, 2007; Liqun, 2007; Fujita, 2008; Filipin *et al.*, 2012; Gopalan and Pandichelvi, 2011; Fujita and Togbe, 2011; Gopalan and Srividhya, 2012; Gopalan and Srividhya, 2012; Filipin, 2005; Gopalan *et al.*, 2014; Gopalan *et al.*, 2014 and Pandichelvi, 2011) for an extensive review of various problem on Diophantine triples. In (Meena *et al.*, Gopalan *et al.*, *and* Gopalan *et al.,)*, special mention is provided because it differs from the earlier one. This paper aims at constructing an interesting family of 3-tuples different from the earlier one. The interesting triple is constructed where the product of any two members of the triple with addition of the same members is a perfect square.

MATERIALS AND METHODS

Let $a = \alpha^2, b = (\alpha^2+1)k^2 + 2\alpha k$ be such that

$$ab + a + b = [(\alpha^2+1)k + \alpha]^2$$

Let c be any non zero integer such that

$$\begin{aligned} ac + a + c &= (\alpha^2+1)c + \alpha^2 = p^2 \\ bc + b + c &= [(\alpha^2+1)k^2 + 2k\alpha + 1]c + (\alpha^2+1)k^2 + 2\alpha k = q^2 \end{aligned} \quad (1)$$

Using some algebra,

$$[(\alpha^2+1)k^2 + 2k\alpha + 1]p^2 - (\alpha^2+1)q^2 = \alpha^2 - (\alpha^2+1)k^2 - 2\alpha k \quad (2)$$

Introducing the linear transformations

$$\begin{aligned} p &= X + (\alpha^2+1)T \\ q &= X + [(\alpha^2+1)k^2 + 2\alpha k + 1]T \end{aligned} \quad (3)$$

in (2), we have $X^2 = [(\alpha^2+1)^2 k^2 + 2\alpha k(\alpha^2+1) + \alpha^2 + 1]T^2 - 1$ (4)

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which is in the form of a Pell equation.
Let $T_0 = 1, X_0 = (\alpha^2 + 1)k + \alpha$ be the initial solutions of (4).
From (3), $p = (\alpha^2 + 1)(k + 1) + \alpha$
From (1), $c = (\alpha^2 + 1)(k + 1)^2 + 2\alpha(k + 1)$
Hence $(\alpha^2, (\alpha^2 + 1)k^2 + 2\alpha k, (\alpha^2 + 1)(k + 1)^2 + 2\alpha(k + 1))$ is the interesting 3-tuple satisfying the required property. Repeating the above process, one can generate many 3-tuples satisfying the required property. For illustration, a few generated triples are given below.

$$((\alpha^2 + 1)k^2 + 2\alpha k, (\alpha^2 + 1)(k + 1)^2 + 2\alpha(k + 1), 4(\alpha^2 + 1)k^2 + 4(\alpha + 1)^2 + (\alpha + 2)^2),$$
$$(1, 2k^2 + 2k, 2(k + 1)(k + 2)), (2k^2 + 2k, 2k^2 + 6k + 4, 8k^2 + 16k + 9),$$
$$(2k^2 + 6k + 4, 8k^2 + 16k + 9, 18k^2 + 42k + 28), (8k^2 + 16k + 9, 18k^2 + 42k + 28, 50k^2 + 110k + 72),$$
$$(4, 5k^2 - 4k, 5k^2 + 6k + 1), (5k^2 - 4k, 5k^2 + 6k + 1, 20k^2 + 4k) \ \textit{and} \ (5k^2 + 6k + 1, 20k^2 + 4k, 45k^2 + 24k + 4)$$

Some numerical examples are presented below
(1, 12, 24), (4, 9, 28), (52, 208, 313), (84, 177, 508), (177, 304, 948) and (264, 456, 1417)

Remark -1:
Replacing k by a Gaussian integer a+ib in each of the above triples, it is noted that each resulting triple is a Gaussian triple satisfying the required property.
Eg: (1, 10+10i, 22+14i), (-4+12i, 4+20i, 1+64i), (4, 13+52i, 44+72i), (-4+6i, 7+16i, 4+44i) and (25+32i, 70+78i, 182+210i).

Remark -2:
In a similar manner, one can generate many special 3-tuples such that the product of any two member of the set minus the sum of the same members is a perfect square. For illustration, a few examples are given below.
(2, 27, 38), (6, 11, 30), (11, 18, 54), (27, 86, 206), (6, 59, 98), (147, 206, 698), (6, 14, 35) and (66, 107, 338)
(3, 4+6i, 12+10i), (12+10i, 24+14i, 67+48i), (2, 7+12i, 14+16i) and (-2+6i, 9+16i, 6+44i)

CONCLUSION
To conclude, one may search for other triples consisting of other forms of special numbers, namely polygonal and centered polygonal numbers.

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