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Research Article

ANALYSIS OF A 2-OUT-OF-2: G SYSTEM WITH SINGLE COLD STANDBY UNIT UNDER PREVENTIVE MAINTENANCE SUBJECT TO MAXIMUM REPAIR TIME

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ABSTRACT
A 2-out-of-2: G system with single unit in cold standby has been analyzed stochastically with the concepts of preventive maintenance and maximum repair time. The faults which occur during operating period of the system have been rectified by a single server with immediate visit whenever needed. The system undergoes for preventive maintenance after a pre-specific time of operation up to which no failure occurred in the system. The failed unit is replaced by new one in case its repair is not possible by the server in a given fixed repair time (called maximum repair time). The repair activities are perfect. The random variables associated with failure time of the unit and different repair activities are independent to each other and follow Weibull distribution. The expressions for some important reliability measures of the system model are derived in steady state by adopting the technique of semi-Markov process and regenerative point. The behavior of these measures has been shown numerically for arbitrary values of the parameters and costs. The results for mean time to system failure (MTSF), availability and profit function have also been obtained for particular cases of the Weibull distribution.

Keywords: *2-out-of-2: G System, Single Cold Standby, Replacement, Preventive Maintenance, Maximum Repair Time, Weibull Distribution and Reliability Measures*

INTRODUCTION
The performance of systems vulnerable to damage can be protected by providing better repair mechanisms and configurations of the components. The k-out-of-n: G systems have been considered as the effective structures in such situations for providing reliable and efficient services to the customers. As a result of which some particular cases of these systems have been suggested by the scholars for their use without any trouble. The 2-out-of-2: G systems have been analyzed by the researchers including Kumar *et al.*, (2013) and Permila and Malik (2015) by introducing the concepts of degradation of the unit after repair, arrival time of the server and priority to repair over replacement. The ideas of preventive maintenance and maximum repair time have been used by the researchers such as Gupta *et al.*, (1991), Malik *et al.*, (2009), Malik and Barak (2013) and Munday *et al.*, (2015) while analyzing standby systems. But these concepts have not been used jointly by the researchers in case of 2-out-of-2: G system with single cold standby. Also, in most of the system models the random variables associated with failure time and different repair policies have been considered to follow negative exponential distribution. However, there may be situations where failure time, repair times and others follow Weibull distribution.
In view of these considerations, the purpose of the present paper is to analyze stochastically a 2-out-of-2: G system with single cold standby. The model has been developed by considering the ideas of preventive maintenance and maximum repair time. The faults which occur during operating period of the system have been rectified by a single server with immediate visit whenever needed. The system undergoes for preventive maintenance after a pre-specific time of operation up to which no failure occurred in the system. The failed unit is replaced by new one in case its repair is not possible by the server in a given fixed repair time (called maximum repair time). The repair activities are perfect. The random variables associated with failure time of the unit and different repair activities are independent to each other and follow Weibull distribution. The expressions for some important reliability measures of the system model such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to repair, replacement and preventive maintenance, expected number of

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repairs, replacements and preventive maintenance of the unit and profit function are derived in steady state by adopting the technique of semi-Markov process and regenerative point. The behavior of these measures has been shown numerically for arbitrary values of the parameters and costs. The results for MTSF, availability and profit function have also been obtained for particular cases of the Weibull distribution.

System Description and Assumptions

1. There is a 2-out-of-2: G system with one unit in cold standby.
2. The units are identical in nature having two modes – operative and failure.
3. There is a single server who visits the system immediately
4. The unit undergoes for preventive maintenance after a pre specific operation time ‘t’
5. The failed unit undergoes for replacement when its repair is not possible by the server in a given specific time‘t’**.**
6. The times to failure, preventive maintenance, replacement and repair of the unit are independent random variables which follow Weibull distribution.
7. The failed unit works as new after repair.
8. The random variables are statistically independent.
9. The switch over is instantaneous and perfect.

Notations

E	: Set of regenerative states
O	: Unit is operative
Cs	: Unit in cold standby
f(t)/F(t)	: pdf / cdf of the failure time of the unit
g(t)/G(t)	: pdf / cdf of the repair time of the unit
n(t)/N(t)	: pdf / cdf of the preventive maintenance time of the unit
r(t)/R(t)	: pdf / cdf of the replacement time of the unit
k(t)/K(t)	: pdf / cdf of the rate by which unit undergoes for preventive maintenance
a(t)/A(t)	: pdf / cdf of the rate by which unit undergoes for replacement
FUr/FUR	: Unit is failed and under repair / under repair continuously from previous state
FWr/FWR	: Unit is failed and waiting for repair / waiting for repair continuously from previous state
FURp/FURP	: Unit is failed and under replacement / under replacement continuously from previous state
FWRp/FWRP	: Unit is failed and waiting for replacement / waiting for replacement continuously from previous state
FUPm/FUPM	: Unit is failed and under preventive maintenance / under preventive maintenance continuously from previous state
FWRp/FWRP	: Unit is failed and waiting for preventive maintenance / waiting for preventive maintenance continuously from previous state
$q_{ij}(t)$/ $Q_{ij}(t)$	: pdf / cdf of first passage time from regenerative state Si to a regenerative state Sj or to a failed state Sj without visiting any other regenerative state in (0, t]
$q_{ij.k}$ (t)/$Q_{ij.k}(t)$	: pdf/cdf of direct transition time from regenerative state Si to a regenerative state Sj or to a failed state Sj visiting state S_k once in (0, t]
$M_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	: Probability that the server is busy in the state S_i up to time ‘t’ without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i	: The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$, Where, T denotes the time to system failure.

m_{ij} : Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that

$$\mu_i = \sum_j m_{ij} \text{ and } m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$$

Ⓢ/© : Notation for Laplace-Stieltjes convolution/Laplace convolution

*/** : Notation for Laplace Transformation (LT)/Laplace Stieltjes Transformation (LST)

State Transition Diagram

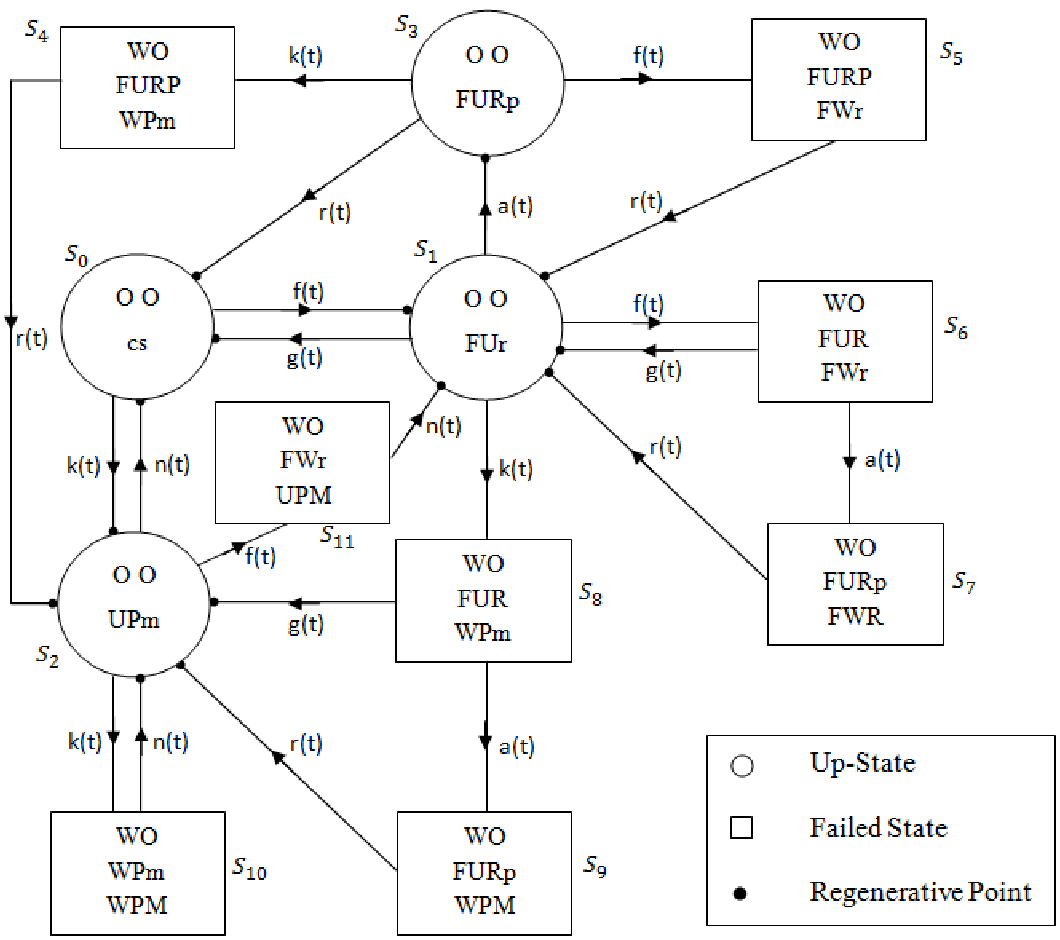

Figure 1: State Transition Diagram

The transition states S_0 , S_1, S_2 *and* S_3 are regenerative states and S_4, S_5 , S_6 , S_7 , S_8 , S_9 , S_{10} *and* S_{11} are non-regenerative as shown in above figure.

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$

$p_{01} = \int_0^\infty f(t)\overline{K(t)}dt$, $p_{02} = \int_0^\infty k(t)\overline{F(t)}dt$, $p_{10} = \int_0^\infty g(t)\overline{A(t)}\,\overline{F(t)}\,\overline{K(t)}dt$,
$p_{13} = \int_0^\infty a(t)\overline{G(t)}\,\overline{F(t)}\,\overline{K(t)}dt$, $p_{16} = \int_0^\infty f(t)\overline{A(t)}\,\overline{G(t)}\,\overline{K(t)}dt$,
$p_{18} = \int_0^\infty k(t)\overline{A(t)}\,\overline{F(t)}\,\overline{G(t)}dt$, $p_{20} = \int_0^\infty n(t)\overline{K(t)}\,\overline{F(t)}dt$,

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$p_{2,10} = \int_0^\infty k(t)\overline{N(t)}\,\overline{F(t)}dt$, $p_{2,11} = \int_0^\infty f(t)\overline{K(t)}\,\overline{N(t)}dt$
$p_{30} = \int_0^\infty r(t)\overline{F(t)}\,\overline{K(t)}dt$, $p_{34} = \int_0^\infty k(t)\overline{R(t)}\,\overline{F(t)}dt$, $p_{35} = \int_0^\infty f(t)\overline{R(t)}\,\overline{K(t)}dt$
$p_{42} = \int_0^\infty r(t)dt$, $p_{51} = \int_0^\infty r(t)dt$, $p_{61} = \int_0^\infty g(t)\overline{A(t)}dt$ $p_{67} = \int_0^\infty a(t)\overline{G(t)}dt$
$p_{71} = \int_0^\infty r(t)dt$, $p_{82} = \int_0^\infty g(t)\overline{A(t)}dt$, $p_{89} = \int_0^\infty a(t)\overline{G(t)}dt$, $p_{92} = \int_0^\infty r(t)dt$,
$p_{10,2} = \int_0^\infty n(t)dt$, $p_{11,1} = \int_0^\infty n(t)dt$
$p_{11.6} = p_{16}Ⓢp_{61}$, $p_{11.67} = p_{16}Ⓢp_{67}Ⓢp_{71}$, $p_{12.8} = p_{18}Ⓢp_{82}$, $p_{12.89} = p_{18}Ⓢp_{89}Ⓢp_{92}$,
$p_{22.(10)} = p_{2,10}Ⓢp_{10,2}$, $p_{21.(11)} = p_{2,11}Ⓢp_{11,1}$, $p_{31.5} = p_{35}Ⓢp_{51}$, $p_{32.4} = p_{34}Ⓢp_{42}$
For these transition probabilities, it can be verified that
$p_{01} + p_{02} = p_{10} + p_{13} + p_{16} + p_{18} = p_{20} + p_{2,10} + p_{2,11} = p_{30} + p_{34} + p_{35} = p_{42} = p_{51} = p_{61} + p_{67} = p_{71} = p_{82} + p_{89} = p_{92} = p_{10,2} = p_{11,1} = p_{10} + p_{11.6} + p_{11.67} + p_{13} + p_{12.8} + p_{12.89} = p_{20} + p_{22.(10)} + p_{21.(11)} = p_{30} + p_{32.4} + p_{31.5} = 1$

Mean Sojourn Times

The unconditional mean time taken by the system to transit from any state S_i when time is counted from epoch at entrance into state S_j is stated as (Cox, 1962)
$m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$ and the mean sojourn times μ_i in state S_i is given by
$\mu_0 = m_{01} + m_{02}$, $\mu_1 = m_{10} + m_{13} + m_{16} + m_{18}$, $\mu_2 = m_{20} + m_{2,10} + m_{2,11}$
$\mu_3 = m_{30} + m_{34} + m_{35}$, $\mu_1' = m_{10} + m_{13} + m_{11.6} + m_{12.8} + m_{11.67} + m_{12.89}$,
$\mu_2' = m_{20} + m_{22.(10)} + m_{21.(11)}$, $\mu_3' = m_{30} + m_{32.4} + m_{31.5}$

Reliability Measures

Mean Time to System Failure (MTSF)

Let $\emptyset_i(t)$ be the cdf of the first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\emptyset_i(t) = \sum_j Q_{ij}(t) \text{ Ⓢ } \emptyset_j(t) + \sum_k Q_{ik}(t) \quad (1)$$

Where, j is an operative regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly.
The system equations given in (1) can be obtained as:

$$\emptyset_0(t) = Q_{01}(t) \text{ Ⓢ } \emptyset_1(t) + Q_{02}(t) \text{ Ⓢ } \emptyset_2(t)$$
$$\emptyset_1(t) = Q_{10}(t) \text{ Ⓢ } \emptyset_0(t) + Q_{13}(t) \text{ Ⓢ } \emptyset_3(t) + Q_{16}(t) + Q_{18}(t)$$
$$\emptyset_2(t) = Q_{20}(t) \text{ Ⓢ } \emptyset_0(t) + Q_{2,10}(t) + Q_{2,11}(t)$$
$$\emptyset_3(t) = Q_{30}(t) \text{ Ⓢ } \emptyset_0(t) + Q_{34}(t) + Q_{35}(t) \quad (2)$$

Taking LST of the relations (2) and solving for $\emptyset_0^{**}(s)$, we get
$R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s}$
The reliability can be obtained by taking inverse Laplace transform of (2) and MTSF is given by

$$MTSF(T_0) = \lim_{s\to 0} R^*(s) = \frac{N_1}{D_1} \quad (3)$$

Where $N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{01}p_{13}\mu_3$ and $D_1 = 1 - p_{01}(p_{10} + p_{13}p_{30}) - p_{02}p_{20}$

Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant 't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as :

$$A_i(t) = M_i(t) + \sum_j q_{ij}^{(n)} © A_j(t) \quad (4)$$

Where, j is any successive regenerative state to which the given regenerative state i can transit through n transitions, and
The system equations given in (4) can be obtained as:

$$A_0(t) = M_0(t) + q_{01}(t)© A_1(t) + q_{02}(t)© A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)© A_0(t) + (q_{11.6}(t) + q_{11.67}(t))© A_1(t) + (q_{12.8}(t) + q_{12.89}(t))© A_2(t) + q_{13}(t)© A_3(t)$$

$A_2(t) = M_2(t) + q_{20}(t)© A_0(t) + q_{21.(11)}(t)© A_1(t) + q_{22.(10)}(t)© A_2(t)$
$A_3(t) = M_3(t) + q_{30}(t)© A_0(t) + q_{31.5}(t)© A_1(t) + q_{32.4}(t)© A_2(t)$ (5)
Where,
$M_0(t) = \int_0^\infty \overline{F(t)}\,\overline{K(t)}dt$, $M_1(t) = \int_0^\infty \overline{A(t)}\,\overline{F(t)}\,\overline{G(t)}\,\overline{K(t)}dt$, $M_2(t) = \int_0^\infty \overline{N(t)}\,\overline{K(t)}dt$
$M_3(t) = \int_0^\infty \overline{R(t)}\,\overline{F(t)}\,\overline{K(t)}dt$
Taking LT of relations (5) and solving for $A_0^*(s)$, we get the steady-state availability as
$A_0(\infty) = \lim_{s\to 0} s A_0^*(s) = \frac{N_2}{D_2}$ (6)
Where,
$N_2 = \mu_0\{(1 - p_{22.(10)})(p_{10} + p_{13}p_{30}) + p_{20}(p_{12.8} + p_{12.89} + p_{13}p_{32.4})\} + \mu_1\{p_{01}p_{20} + p_{21.(11)}\} +$
$\mu_2\{(1 - p_{11.6} - p_{11.67} - p_{13}p_{31.5}) - p_{01}(p_{10} + p_{13}p_{30})\} + p_{13}\mu_3\{p_{01}(1 - p_{22.(10)}) +$
$p_{02}(p_{13}p_{21.(11)})\}$
and $D_2 = \mu_0\{(1 - p_{22.(10)})(p_{10} + p_{13}p_{30}) + p_{20}(p_{12.8} + p_{12.89} + p_{13}p_{32.4})\} + \mu_1'\{p_{21.(11)} + p_{20}p_{01}\} +$
$\mu_2'\{(1 - p_{11.6} - p_{11.67} - p_{13}p_{31.5}) - p_{01}(p_{10} + p_{13}p_{30})\} + \mu_3'\{p_{13}(p_{21.(11)} + p_{01}p_{20})\}$

Busy Period Analysis of the Server

Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$.The recursive relations for $B_i^R(t)$ are given as:
$B_i^R(t) = W_i^R(t) + \sum_j q_{ij}^{(n)}(t)© B_j^R(t)$ (7)
Where, j is a subsequent regenerative state to which state i transits through n transitions.
The system equations given in (7) can be obtained as:
$B_0^R(t) = q_{01}(t)© B_1^R(t) + q_{02}(t)© B_2^R(t)$
$B_1^R(t) = W_1^R(t) + q_{10}(t)© B_0^R(t) + (q_{11.6}(t) + q_{11.67}(t))© B_1^R(t)$
$+ (q_{12.8}(t) + q_{12.89}(t))© B_2^R(t) + q_{13}(t)© B_3^R(t)$
$B_2^R(t) = q_{20}(t)© B_0^R(t) + q_{22.(11)}(t)© B_1^R(t) + q_{22.(10)}(t)© B_2^R(t)$
$B_3^R(t) = q_{30}(t)© B_0^R(t) + q_{31.5}(t)© B_1^R(t) + q_{32.4}(t)© B_2^R(t)$ (8)
Where, $W_1^R(t) = \int_0^\infty \overline{G(t)}\,\overline{A(t)}\,\overline{F(t)}\,\overline{K(t)}dt$
Taking LT of relations (8) and solving for $B_0^{R^*}(s)$. In the long run, the time for which the system is under repair is given by

$B_0^R(\infty) = \lim_{s\to 0} s B_0^{R^*}(s) = \frac{N_3}{D_2}$ (9)

Where $N_3 = \mu_1\{p_{01}(1 - p_{21.(11)} + p_{22.(10)}) + p_{21.(11)}\}$ and D_2 is already specified

Due to Replacement

Let $B_i^{Rp}(t)$ be the probability that the sever is busy in replacing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $B_i^{Rp}(t)$ are given by:
$B_i^{Rp}(t) = W_i^{Rp}(t) + \sum_j q_{ij}^{(n)}(t)© B_j^{Rp}(t)$ (10)
Where, j is a subsequent regenerative state to which state i transits through n transitions.
The system equations given in (10) can be obtained as:
$B_0^{Rp}(t) = q_{01}(t)©B_1^{Rp}(t) + q_{02}(t)©B_2^{Rp}(t)$
$B_1^{Rp}(t) = q_{10}(t)©B_0^{Rp}(t) + (q_{11.6}(t) + q_{11.67}(t))© B_1^{Rp}(t)$
$+(q_{12.8}(t) + q_{12.89}(t))© B_2^{Rp}(t) + q_{13}(t)© B_3^{Rp}(t)$
$B_2^{Rp}(t) = q_{20}(t)©B_0^{Rp}(t) + q_{21.(11)}(t)© B_1^{Rp}(t) + q_{22.(10)}(t)© B_2^{Rp}(t)$
$B_3^{Rp}(t) = W_3^{Rp}(t) + q_{30}(t)©B_0^{Rp}(t) + q_{31.5}(t)©B_1^{Rp}(t) + q_{32.4}(t)©B_2^{Rp}(t)$ (11)
Where, $W_3^{Rp}(t) = \int_0^\infty \overline{R(t)}\,\overline{K(t)}\,\overline{F(t)}\,dt$

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Taking LT of relations (11) and solving for $B_0^{Rp*}(s)$, we get the time for which the system is under replacement is given by

$$B_0^{Rp} = \lim_{s\to 0} s\, B_0^{Rp*}(s) = \frac{N_4}{D_2} \quad (12)$$

Where, $N_4 = \mu_3\{p_{13}(p_{21.(11)} + p_{01}p_{20})\}$ and D_2 is already specified.

Due to Preventive Maintenance

Let $B_i^{Pm}(t)$ be the probability that the sever is busy in preventive maintenance of the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $B_i^{Pm}(t)$ are given by:

$$B_i^{Pm}(t) = W_i^{Pm}(t) + \sum_j q_{ij}^{(n)}(t) © B_j^{Pm}(t) \quad (13)$$

Where, j is a subsequent regenerative state to which state i transits through n transitions.

The system equations given in (13) can be obtained as:

$$B_0^{Pm}(t) = q_{01}(t) © B_1^{Pm}(t) + q_{02}(t) © B_2^{Pm}(t)$$
$$B_1^{Pm}(t) = W_2^{Pm}(t) + q_{10}(t) © B_0^{Pm}(t) + (q_{11.6}(t) + q_{11.67}(t)) © B_1^{Pm}(t)$$
$$+(q_{12.8}(t) + q_{12.89}(t)) © B_2^{Pm}(t) + q_{13}(t) © B_3^{Pm}(t)$$
$$B_2^{Pm}(t) = q_{20}(t) © B_0^{Pm}(t) + q_{21.(11)}(t) © B_1^{Pm}(t) + q_{22.(10)}(t) © B_2^{Pm}(t)$$
$$B_3^{Pm}(t) = q_{30}(t) © B_0^{Pm}(t) + q_{31.5}(t) © B_1^{Pm}(t) + q_{32.4}(t) © B_2^{Pm}(t) \quad (14)$$

Where, $W_2^{Pm}(t) = \int_0^\infty \overline{N(t)}\,\overline{K(t)}\,\overline{F(t)}\,dt$

Taking LT of relations (14) and solving for $B_0^{Pm*}(s)$, we get the time for which the system is under replacement is given by

$$B_0^{Pm} = \lim_{s\to 0} s\, B_0^{Pm*}(s) = \frac{N_5}{D_2} \quad (15)$$

Where,

$N_5 = \mu_2\{(1 - p_{11.6} - p_{11.67} - p_{13}p_{31.5}) - p_{01}(p_{13}p_{30} + p_{10})\}$ and D_2 is already specified.

Expected Number of Visit by Server

Due to Repairs

Let $R_i(t)$ be the expected number of repairs of the unit in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $R_i(t)$ are given by:

$$R_i(t) = \sum_j Q_{ij}(t) Ⓢ \left(\delta_j + R_j(t)\right) \quad (16)$$

Where, j is any regenerative state to which the given regenerative state i transits and δ_j=1, if j is the regenerative state where the server does job afresh otherwise δ_j=0.

The system equations given in (16) can be obtained as:

$$R_0(t) = Q_{01}(t) Ⓢ R_1(t) + Q_{02}(t) Ⓢ R_2(t)$$
$$R_1(t) = Q_{10}(t) Ⓢ (1 + R_0(t)) + Q_{11.6}(t) Ⓢ (1 + R_1(t)) + Q_{11.67}(t) Ⓢ R_1(t)$$
$$+Q_{12.8}(t) Ⓢ (1 + R_2(t)) + Q_{12.89}(t) Ⓢ R_2(t) + Q_{13}(t) Ⓢ R_3(t)$$
$$R_2(t) = Q_{20}(t) Ⓢ R_0(t) + Q_{21.(11)}(t) Ⓢ R_1(t) + Q_{22.(10)}(t) Ⓢ R_2(t)$$
$$R_3(t) = Q_{30}(t) Ⓢ R_0(t) + Q_{31.5}(t) Ⓢ R_1(t) + Q_{32.4}(t) Ⓢ R_2(t) \quad (17)$$

Taking LST of relations (17) and solving for $R_0^{**}(s)$, we get the expected number of repairs of the unit per unit time as

$$R_0 = \lim_{s\to 0} s\, R_0^{**}(s) = \frac{N_6}{D_2} \quad (18)$$

Where $N_6 = (p_{11.6} + p_{12.8})(p_{01}p_{20} + p_{21.(11)})$ and D_2 is already specified.

Due to Replacements

Let $Rp_i(t)$ be the expected number of replacements of the unit in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $Rp_i(t)$ are given by:

$$Rp_i(t) = \sum_j Q_{ij}(t) Ⓢ \left(\delta_j + Rp_j(t)\right) \quad (19)$$

Where, j is any regenerative state to which the given regenerative state i transits and δ_j=1, if j is the regenerative state where the server does job afresh otherwise δ_j=0.
The system equations given in (19) can be obtained as:

$$Rp_0(t) = Q_{01}(t)Ⓢ Rp_1(t) + Q_{02}(t)Ⓢ Rp_2(t)$$

$$Rp_1(t) = Q_{10}(t)Ⓢ Rp_0(t) + Q_{11.6}(t)Ⓢ Rp_1(t) + Q_{11.67}(t)Ⓢ (1 + Rp_1(t))$$
$$+Q_{12.8}(t)Ⓢ Rp_2(t) + Q_{12.89}(t)Ⓢ (1 + Rp_2(t)) + Q_{13}(t)Ⓢ Rp_3(t)$$

$$Rp_2(t) = Q_{20}(t)Ⓢ Rp_0(t) + Q_{21.(11)}(t)Ⓢ Rp_1(t) + Q_{22.(10)}(t)Ⓢ Rp_2(t)$$

$$Rp_3(t) = Q_{30}(t)Ⓢ \left(1 + Rp_0(t)\right) + Q_{31.5}(t)Ⓢ \left(1 + Rp_1(t)\right) + Q_{32.4}(t)Ⓢ (1 + Rp_2(t)) \qquad (20)$$

Taking LST of relations (20) and solving for $Rp_0^{**}(s)$, we get the expected number of replacements of the unit per unit time as

$$Rp_0 = \lim_{s\to 0} s\, Rp_0^{**}(s) = \frac{N_7}{D_2} \qquad (21)$$

Where, $N_7 = (p_{11.67} + p_{12.89} + p_{13})\left(p_{20}p_{01} + p_{21.(11)}\right)$ and D_2 is already specified.

Due to Preventive Maintenances

Let $Pm_i(t)$ be the expected number of preventive maintenances of the unit in $(0,t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relations for $Pm_i(t)$ are given by:

$$Pm_i(t) = \sum_j Q_{ij}(t)Ⓢ \left(\delta_j + Pm_j(t)\right) \qquad (22)$$

Where, j is any regenerative state to which the given regenerative state i transits and δ_j=1, if j is the regenerative state where the server does job afresh otherwise δ_j=0.
The system equations given in (22) can be obtained as:

$$Pm_0(t) = Q_{01}(t)Ⓢ Pm_1(t) + Q_{021}(t)Ⓢ Pm_2(t)$$

$$Pm_1(t) = Q_{10}(t)Ⓢ Pm_0(t) + (Q_{11.6}(t) + Q_{11.67}(t))Ⓢ Pm_1(t)$$
$$+(Q_{12.8}(t) + Q_{12.89}(t))Ⓢ Pm_2(t) + Q_{13}(t)Ⓢ Pm_3(t)$$

$$Pm_2(t) = Q_{20}(t)Ⓢ \left(1 + Pm_0(t)\right) + Q_{21.(11)}(t)Ⓢ \left(1 + Pm_1(t)\right) + Q_{22.(10)}(t)Ⓢ (1 + Pm_2(t))$$

$$Pm_3(t) = Q_{30}(t)Ⓢ Pm_0(t) + Q_{31.5}(t)Ⓢ Pm_1(t) + Q_{32.4}(t)Ⓢ Pm_2(t) \qquad (23)$$

Taking LST of relations (23) and solving for $Pm_0^{**}(s)$, we get the expected number of preventive maintenances of the unit per unit time as

$$Pm_0 = \lim_{s\to 0} s\, Pm_0^{**}(s) = \frac{N_8}{D_2} \qquad (24)$$

Where $N_8 = (1 - p_{11.6} - p_{11.67} - p_{13}p_{31.5} - p_{01}(p_{10} + p_{13}p_{30}))$ and D_2 is already specified.

Cost-Benefit Analysis

The profit function in the time interval t is given by:-
P(t) = Expected revenue in (0,t] – expected total cost in (0,t]
In view of the various costs, the profit earned to the system model in steady state can be attained as:

$$P = K_0A_0 - K_1B_0^R - K_2B_0^{Rp} - K_3R_0 - K_4Rp_o - K_5B_0^{Pm} - K_6Pm_o \qquad (25)$$

Where,

K_0 = Revenue per unit up-time of the system.
K_1 =Cost per unit time for which server is busy due to repair.
K_2 =Cost per unit time for which server is busy due to replacement.
K_3 =Cost per unit time repair.
K_4 =Cost per unit time replacement.
K_5 =Cost per unit time for which server is busy due to preventive maintenance.
K_6 =Cost per unit time preventive maintenance.

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Tabulation of the Performance Measures

Table 1: MTSF Vs Failure Rate

Rayleigh(R) Distribution							**Exponential(E) Distribution**					
2λ	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**
0.02	42.63404	44.7977	43.3927	43.3025	31.6515	48.5524	68.65767	72.2366	69.81192	69.79823	44.62517	78.71961
0.04	33.49406	36.1542	34.4198	34.2984	26.1775	36.97	50.39554	54.49637	51.68626	51.6811	35.03438	55.9265
0.06	27.58686	30.2576	28.5157	28.3843	22.3461	29.8647	39.12879	42.99416	40.32566	40.33048	28.55101	42.54338
0.08	23.46067	25.9893	24.3427	24.2102	19.515	25.0657	31.5907	35.04708	32.64845	32.6613	23.91483	33.86955
0.1	20.41916	22.7635	21.241	21.1114	17.3381	21.6101	26.25064	29.2922	27.17328	27.19202	20.45819	27.85939
0.12	18.08665	20.2443	18.8479	18.7228	15.6127	19.0052	22.30275	24.97082	23.10655	23.12948	17.79628	23.48777
0.14	16.24267	18.2252	16.9472	16.8273	14.2118	16.9728	19.28556	21.63061	19.98817	20.01399	15.69268	20.18807
0.16	14.74935	16.5728	15.4023	15.2878	13.052	15.3438	16.91746	18.98701	17.53473	17.56251	13.9947	17.62354
0.18	13.51607	15.1968	14.1228	14.0135	12.0762	14.0097	15.01774	16.85308	15.56309	15.59214	12.59966	15.58252

Table 2: Availability Vs Failure Rate

	Rayleigh(R) Distribution						Exponential(E) Distribution					
2λ	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**
0.02	0.97308	0.97618	0.97439	0.97415	0.96498	0.97716	0.988831	0.991005	0.989622	0.989453	0.983681	0.990845
0.04	0.961784	0.96791	0.96439	0.96388	0.9532	0.96581	0.981948	0.986629	0.983646	0.983286	0.975595	0.984141
0.06	0.950428	0.9595	0.9543	0.95353	0.94149	0.95441	0.974242	0.981711	0.976942	0.976374	0.966823	0.976599
0.08	0.939062	0.95098	0.94416	0.94313	0.92987	0.94301	0.965806	0.976293	0.969584	0.968796	0.957448	0.968312
0.1	0.927728	0.9424	0.93402	0.93273	0.91837	0.93164	0.956727	0.970417	0.961643	0.960625	0.947552	0.95937
0.12	0.916459	0.93378	0.92391	0.92235	0.90701	0.92033	0.94709	0.964123	0.953184	0.951933	0.937208	0.949854
0.14	0.905284	0.92515	0.91385	0.91204	0.8958	0.90912	0.936971	0.95745	0.944271	0.942784	0.926486	0.939843
0.16	0.894226	0.91654	0.90386	0.9018	0.88475	0.89803	0.926444	0.950434	0.934964	0.933241	0.915451	0.929411
0.18	0.883304	0.90795	0.89396	0.89167	0.87388	0.88708	0.915575	0.943111	0.925318	0.923361	0.90416	0.918624

Table 3: Profit Vs Failure Rate

Rayleigh(R) Distribution							**Exponential(E) Distribution**					
2λ	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**	**θ=1,β=0.8, α=0.4, δ=0.15,μ=2**	**θ=2.2**	**β=1.2**	**α=0.5**	**δ=0.2**	**μ=2.5**
0.02	4786.13	4803.39	4792.88	4792.58	4742.04	4808.39	4906.278	4918.818	4910.87	4910.228	4873.147	4918.194
0.04	4712.2	4745.77	4725.51	4724.63	4668.15	4733.82	4859.872	4886.396	4869.555	4868.165	4821.338	4872.604
0.06	4640.73	4689.77	4660.39	4658.75	4596.64	4661.83	4809.908	4851.597	4825.077	4822.856	4766.638	4823.379
0.08	4571.33	4635.1	4597.12	4594.61	4527.27	4592.01	4756.845	4814.638	4777.803	4774.688	4709.463	4770.981
0.1	4503.77	4581.57	4535.49	4532.03	4459.84	4524.09	4701.116	4775.726	4728.078	4724.027	4650.203	4715.844
0.12	4437.92	4529.07	4475.32	4470.87	4394.23	4457.92	4643.126	4735.062	4676.231	4671.217	4589.219	4658.376
0.14	4373.67	4477.51	4416.52	4411.07	4330.34	4393.38	4583.253	4692.835	4622.569	4616.58	4526.842	4598.955
0.16	4310.96	4426.85	4359.02	4352.57	4268.09	4330.4	4521.842	4649.227	4567.377	4560.414	4463.378	4537.934
0.18	4249.73	4377.06	4302.77	4295.32	4207.42	4268.91	4459.214	4604.408	4510.923	4502.996	4399.104	4475.634

Conclusion

The numerical behavior of some important reliability measures of the system model has been observed for arbitrary values of the parameters including K_0=5000, K_1=450, K_2=350, K_3=250, K_4=200, K_5=150, K_6=100 as shown in tables 1, 2 and 3. It is analyzed that availability, MTSF, profit decrease with the increase of the failure rate (λ) and the rate by which unit undergoes for preventive maintenance (δ) while their values increase with the increase of repair rate (α), replacement rate (β), maximum repair rate (θ) and preventive maintenance rate (μ). Also, it is clear from the tables that under the stated conditions, the performance measures of the system have more values in case random variables follow Exponential distribution rather than Rayleigh distribution.

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