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Research Article

FUZZY STEADY STATE ANALYSIS OF $M^X/M^{(A,B)}/1$ QUEUE MODELS WITH RANDOM BREAKDOWNS

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ABSTRACT
In this paper we are considering a single server bulk queuing system FM*/FM(a, b)/1 in which the service facility suffers time homogeneous random breakdowns from time to time. In this model repair times are assumed to be exponential. We will try to obtain the Probability Generating Function (PGF) of queue size and system size, the average queue size and average system size and average waiting time in queue and system when the arrival rate, service rate and repair time are fuzzy numbers. A numerical example is also given, which is solved using non-linear parametric programming (NLP).

Key Words: *$M^x/M^{(a,b)}/1$ Queuing System, With Batch Arrivals, Batch Service, Random Breakdowns, Steady State Analysis*

INTRODUCTION
It may happen in several situations that the server is unavailable to the customer over occasional periods of time. The server may then be doing other work, such as maintenance work or servicing secondary customers. The periods for which the server is unavailable are said to be server vacation periods. Systems with server vacations can be used as mode of many production, communication and computer systems. Vacation models have of late received much attention of their interesting theoretical properties as well as for applicability in more complicated queuing models. During servers vacation period or the repair time of the service facility the units or customers have to wait until the sever returns to the system or system becomes operable again. Consequently, such vacations or breakdowns have a definite effect on the system, particularly on the queue length and customer's waiting time in the system Gaver ; Levy and Yechilai ; Fuhrman, Doshi, Keilson and Servi, Cramer, Choudhury and Borthakur, Madan and Madan and Saleh are a few among many authors who have studied queues with server vacations. When server vacations are matter of policy, there could be random breakdowns which are beyond the control or server. Sengupta, Lie et al and Takine and Sengupta studied some queuing system with server breakdown. However this paper deals with a bulk queues $M^x/M^{(a,b)}/1$ with random breakdowns in which the arrival rate, service rate and repair time are fuzzy numbers.
In our system $FM^x/FM^{(a,b)}/1$ we assume that the customers arrive at the system in batches of variable size and are serviced in batches of variable size with a minimum batch size 'a' and a maximum batch size 'b'. Such models may find applications in transportation systems, computer and communication system, etc. Here we consider the model in which the repair time is exponential with random breakdowns. We further assume that the breakdowns are random and time – homogeneous, which means that the service facility may fail not only while it is working but is may fail even when it is idle.

MODEL DESCRIPTION
Consider the $FM^x/FM^{(a,b)}/1$ model with random breakdowns. The arrival process is assumed to be Compound Poisson in which customers arrive in batches of size i with arrival rate $\tilde{\lambda}_{C_i}$ where $0 \le C_i \le 1$, $\sum_{i=1}^{\infty} C_i = 1$ and $\tilde{\lambda} > 0$ is the mean arrival rate of batches which is a fuzzy number. The service rate of a

batch is exponential with mean service rate $\frac{1}{\tilde{\mu}}$ $(\tilde{\mu} > 0)$ which is also a fuzzy number. We also assume that if there are less than 'a' customers in the system the server does not work and there are more than 'b' customers in the system the server start service by taking a batch of 'b' customers only. The probability that the server will breakdown during the interval (t, t+dt, is $\tilde{\beta}dt$) where $\tilde{\beta}$ is a fuzzy number. Repair time is exponential with mean repair time $\frac{1}{\tilde{\gamma}}$ $(\tilde{\gamma} > 0)$ where $\tilde{\gamma}$ is a fuzzy number. We also assume that all the variables are independent.

We define the following
$W_n(t)$ = Pr[at time t there are n ≥ 0 customers in the system and the service channel in the operating state]
$F_n(t)$ = Pr[at time t there are n ≥ 0 customers in the system and service channel is in the failed state ie, under repair]
$P_n(t)$ = Pr[at time t there are n ≥ 0 customers in the system without regard to whether the service channel is in the operating or failed state]

The comparing the system at time t+dt with time t we have the following set of difference differential equations

$$W_n'(t) = \left(\tilde{\lambda} + \tilde{\mu} + \tilde{\beta}\right) W_n(t) + \sum_{i=1}^{n} \tilde{\lambda} C_i \, . \, W_{n-i}(t) + \sum_{i=a}^{b} \tilde{\mu} W_{n+i}(t) + \tilde{\gamma} F_n \;\; ; \; n \geq a \qquad \ldots (1)$$

$$W_n'(t) = -\left(\tilde{\lambda} + \tilde{\beta}\right) W_n(t) + \sum_{i=1}^{n} \tilde{\lambda} C_i . W_{n-i}(t) + \sum_{i=a}^{b} \tilde{\mu} W_{n+i}(t) + \tilde{\gamma} F_n \;\; ; \; 1 \leq n \leq a\text{-}1 \qquad \ldots (2)$$

$$W_0'(t) = -\left(\tilde{\lambda} + \tilde{\beta}\right) W_0(t) + \sum_{i=a}^{b} \tilde{\mu} W_i(t) + \tilde{\gamma} F_0(t) \qquad \ldots (3)$$

$$F_n'(t) = -\left(\tilde{\lambda} + \tilde{\gamma}\right) F_n(t) + \tilde{\beta} W_n(t) + \sum_{i=1}^{n} \tilde{\lambda} C_i F_{n-i}(t) \; ; \; n \geq 1 \qquad \ldots (4)$$

$$F_0'(t) = -\left(\tilde{\lambda} + \tilde{\gamma}\right) F_0(t) + \tilde{\beta} W_0(t) \qquad \ldots (5)$$

STEADY STATE ANALYSIS OF SYSTEM SIZE

Assume that the steady state exists and let $\lim_{t \to \infty} W_n(t) = W_n$, $\lim_{t \to \infty} F_n(t) = F_n$ and $\lim_{t \to \infty} P_n(t) = P_n$ be the corresponding steady state probabilities.
Thus we have the following steady state system equations.

$$\left(\tilde{\lambda} + \tilde{\mu} + \tilde{\beta}\right) W_n = \sum_{i=1}^{n} \tilde{\lambda} C_i \, . \, W_{n-i} + \sum_{i=a}^{b} \tilde{\mu} W_{n+i} + \tilde{\gamma} F_n \;\; ; \; n \geq a \qquad \ldots (6)$$

$$\left(\tilde{\lambda} + \tilde{\beta}\right) W_n = \sum_{i=1}^{n} \tilde{\lambda} C_i . W_{n-i} + \sum_{i=a}^{b} \tilde{\mu} W_{n+i} + \tilde{\gamma} F_n \;\; ; \; 1 \leq n \leq a \qquad \ldots (7)$$

$$\left(\tilde{\lambda}+\tilde{\beta}\right)W_0 = \sum_{i=a}^{b}\tilde{\mu}W_i + \tilde{\gamma}F_0 \qquad ...(8)$$

$$\left(\tilde{\lambda}+\tilde{\gamma}\right)F_n = \tilde{\beta}W_n + \sum_{i=1}^{n}\tilde{\lambda}C_i.F_{n-i} \ ; \ n \geq 1 \qquad ...(9)$$

$$\left(\tilde{\lambda}+\tilde{\gamma}\right)F_0 = \tilde{\beta}W_0 \qquad ...(10)$$

We will the PGFs as follows

$$W(z) = \sum_{n=0}^{\infty}W_nZ^n,\ F(Z) = \sum_{n=0}^{\infty}F_nZ^n,\ P(Z) = \sum_{n=0}^{\infty}P_nZ^n\ C(Z) = \sum_{n=1}^{\infty}C_nZ^n,\ |Z| < 1 \qquad ...(11)$$

Now multiply (6) and (7) by Z^{n+b} and take summation over n from 1 to ∞ ; we have

$$Z^b\left(\tilde{\lambda}+\tilde{\mu}+\tilde{\beta}\right)\sum_{n=1}^{\infty}W_nZ^n = Z^b\tilde{\lambda}\sum_{n=0}^{\infty}\sum_{i=1}^{n}C_iW_{n-i}Z^n + \tilde{\mu}\sum_{n=1}^{\infty}\sum_{i=a}^{b}W_{n-i}Z^{n+b} + Z^b\tilde{\gamma}\sum_{n=1}^{\infty}F_nZ^n;\ n \geq a \qquad ...(12)$$

$$Z^b\left(\tilde{\lambda}+\tilde{\beta}\right)\sum_{n=1}^{\infty}W_nZ^n = Z^b\tilde{\lambda}\sum_{n=1}^{\infty}\sum_{i=1}^{n}C_iW_{n-i}Z^n + Z^b\tilde{\mu}\sum_{n=1}^{\infty}\sum_{i=a}^{b}W_{n-i}Z^{n+b} + Z^b\tilde{\gamma}\sum_{n=1}^{\infty}F_nZ^n;\ 1 \leq n \leq a\text{-}1 \qquad ...(13)$$

Multiply (8) by Z^b and (9) by Z^n and taking $\sum_{n=1}^{\infty}$

$$Z^b\left(\tilde{\lambda}+\tilde{\beta}\right)W_0 = \tilde{\mu}\sum_{i=a}^{b}W_iZ^b + Z^b\tilde{\gamma}F_0 \qquad ...(14)$$

$$\left(\tilde{\lambda}+\tilde{\gamma}\right)\sum_{n=1}^{\infty}F_nZ^n = \tilde{\beta}\sum_{n=1}^{\infty}W_nZ^n + \tilde{\lambda}\sum_{n=1}^{\infty}\sum_{i=1}^{n}C_iF_{n-i}Z^n \ ;\ n \geq 1 \qquad ...(15)$$

Now note that $\sum_{n=1}^{\infty}\sum_{i=1}^{n}C_iW_{n-i}Z^n = W(Z)\ C(Z)$ and

$$\sum_{n=0}^{\infty}\sum_{i=a}^{b}W_{n-i}Z^{n+b} = W(Z)\sum_{i=a}^{b}Z^{b-i} - \sum_{j=a}^{b}Z^{b-j}\sum_{i=0}^{j-1}W_iZ^{-1} \qquad ...(17)$$

Then we add (12), (13) and (14) and use (11), (16) and (17). We will get on simplification

$$W(Z) = \frac{Z^b\tilde{\mu}\sum_{n=0}^{a-1}W_nZ^n - \tilde{\mu}\sum_{j=a}^{b}Z^{b-j}\sum_{i=0}^{j-1}W_iZ^i + Z^b\tilde{\gamma}F(Z)}{Z^b\left(\tilde{\lambda}-\tilde{\lambda}(Z)+\tilde{\mu}+\tilde{\beta}\right) - \tilde{\mu}\sum_{i=a}^{b}Z^{b-i}} \qquad ...(18)$$

Now we add (10) and (15) and use (11)
On simplification we get

$$F(Z) = \frac{\tilde{\beta}W(Z)}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}} \qquad \text{... (19)}$$

Substituting (19) in (18) we get

$$W(Z) = \frac{Z^b\tilde{\mu}\sum_{n=0}^{a-1} W_n Z^n - \tilde{\mu}\sum_{j=a}^{b} Z^{b-j}\sum_{i=0}^{j-1} W_i Z^i}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\gamma}\tilde{\beta}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}\sum_{i=a}^{b} Z^{b-i}} \qquad \text{... (20)}$$

Substituting (20) in (19) we get

$$F(Z) = \frac{\frac{\tilde{\beta}W(Z)}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\left[Z^b\tilde{\mu}\sum_{n=0}^{a-1} W_n Z^n - \tilde{\mu}\sum_{j=a}^{b} Z^{b-j}\sum_{i=0}^{j-1} W_i Z^i\right]}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}\sum_{i=a}^{b} Z^{b-i}} \qquad \text{... (21)}$$

Now adding (20) in (21) we get

$$P(Z) = \frac{\left[1 + \frac{\tilde{\beta}W(Z)}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right]\left[Z^b\tilde{\mu}\sum_{n=0}^{a-1} W_n Z^n - \tilde{\mu}\sum_{j=a}^{b} Z^{b-j}\sum_{i=0}^{j-1} W_i Z^i\right]}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}\sum_{i=a}^{b} Z^{b-i}} \qquad \text{... (22)}$$

By Rouche's Theorem the denominator of the RHS of (22) has b zeros within and on the unit circle |Z| = 1. Thus the numerator must vanish for each of these zeros, giving b linear equations in terms of W_i (i = 0 to b) which are sufficient to determine all the b unknowns.

SOME PARTICULAR CASES

If the service facility does not suffer breakdowns then $\tilde{\beta} = 0$ and so F(Z) = 0. Then P(Z) = W(Z) . . . (23) is the PGF of $FM^x / FM^{(a,b)} / 1$ without server breakdowns.

If the server renders service to the customers in batches of fixed size b then with a = b we have

$$W(Z) = \frac{\tilde{\mu}\sum_{i=0}^{b-i} W_i Z^i (Z^0 - 1)}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}} \qquad \text{... (24)}$$

$$F(Z) = \frac{\left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\left[\tilde{\mu}\sum_{i=0}^{a-1} W_n Z^n (Z^b - 1)\right]}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}} \qquad \text{... (25)}$$

$$P(Z) = \frac{\left(1+\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\left[\tilde{\mu}\sum_{i=0}^{a-1} W_n Z^n (Z^b - 1)\right]}{Z^b\left[\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\mu} + \tilde{\beta} - \left(\frac{\tilde{\beta}\,\tilde{\gamma}}{\tilde{\lambda} - \tilde{\lambda}(Z) + \tilde{\gamma}}\right)\right] - \tilde{\mu}} \qquad \text{... (26)}$$

These are the PGFs of FM^x / $FM^{(a,b)}$ / 1 queue with batch arrivals, service in batches of fixed size b, random breakdowns and exponential repairs.

Considering the above queue with single arrivals, one by one exponential service, random breakdowns and exponential repairs ie. we have $C_1 = 1$, $C_i = 0$, $i \neq 1$, $a = b = 1$. Then we have

$$W(Z) = \frac{(Z-1)\,\tilde{\mu} W_0\left(\tilde{\lambda} + \tilde{\gamma} - \tilde{\lambda} Z\right)}{\left(\left(\tilde{\lambda} + \tilde{\mu} + \tilde{\beta}\right)Z - \tilde{\lambda}(Z)^2 - \tilde{\mu}\right)\left(\tilde{\lambda} + \tilde{\gamma} - \tilde{\lambda} Z\right) - \tilde{\beta}} \qquad \text{... (27)}$$

$$F(Z) = \frac{\tilde{\beta}(Z-1)\,\tilde{\mu} W_0}{\left(\left(\tilde{\lambda} + \tilde{\mu} + \tilde{\beta}\right)Z - \tilde{\lambda}(Z)^2 - \tilde{\mu}\right)\left(\tilde{\lambda} + \tilde{\gamma} - \tilde{\lambda} Z\right) - \tilde{\beta}\,\tilde{\gamma}\,Z} \qquad \text{... (28)}$$

$$P(Z) = \frac{(Z-1)\,\tilde{\mu}\left(\left(\tilde{\lambda} + \tilde{\gamma} - \tilde{\lambda} Z\right) + \tilde{\beta}\right) W_0}{\left(\left(\tilde{\lambda} + \tilde{\mu} + \tilde{\beta}\right)Z - \tilde{\lambda}(Z)^2 - \tilde{\mu}\right)\left(\tilde{\lambda} + \tilde{\gamma} - \tilde{\lambda} Z\right) - \tilde{\beta}\,\tilde{\gamma}\,Z} \qquad \text{... (29)}$$

Now for finding W0 we will use the normalizing condition P(1) = 1. However when Z = 1, we get the indeterminate form in (29). So we use L' Hospitals Rule in (27) and (28) and we get

$$W(1) = \lim_{Z\to 1} W(Z) = \frac{\tilde{\gamma}\,\tilde{\mu}\,W_0}{\tilde{\mu}\tilde{\gamma} - \tilde{\lambda}\left(\tilde{\beta} + \tilde{\gamma}\right)} \qquad \text{... (30)}$$

$$F(1) = \lim_{Z\to 1} F(Z) = \frac{\tilde{\beta}\,\tilde{\mu}\,W_0}{\tilde{\mu}\tilde{\gamma} - \tilde{\lambda}\left(\tilde{\beta} + \tilde{\gamma}\right)} \qquad \text{... (31)}$$

(30) and (31) are the steady state probabilities that the service channel is in working state and is under repairs.

$P(1) = W(1) + F(1) = 1$

On simplification we get

$$W_0 = \frac{\tilde{\mu}\,\tilde{\gamma} - \tilde{\lambda}\left(\tilde{\beta} + \tilde{\gamma}\right)}{\tilde{\mu}\left(\tilde{\beta} + \tilde{\gamma}\right)} \qquad \text{... (32)}$$

where $\frac{\tilde{\lambda}\left(\tilde{\beta} + \tilde{\gamma}\right)}{\tilde{\mu}\,\tilde{\gamma}} < 1$

is the steady state condition under steady state solution exist.

The proportion of time the server is busy is given by

$$\rho = W(1) - W_0 \qquad \text{... (33)}$$

Substituting (32) in (29) we get

$$P(Z) = \frac{(Z-1)\,\tilde{\mu}\left(\left(\tilde{\lambda}+\tilde{\gamma}-\tilde{\lambda}Z\right)+\tilde{\beta}\right)\dfrac{\tilde{\mu}\,\tilde{\gamma}-\tilde{\lambda}\left(\tilde{\beta}+\tilde{\gamma}\right)}{\tilde{\mu}\left(\tilde{\beta}+\tilde{\gamma}\right)}}{\left(\left(\tilde{\lambda}+\tilde{\mu}+\tilde{\beta}\right)Z-\tilde{\lambda}(Z)^2-\tilde{\mu}\right)\left(\tilde{\lambda}+\tilde{\gamma}-\tilde{\lambda}Z\right)-\tilde{\beta}\,\tilde{\gamma}\,Z} \quad \text{... (34)}$$

when $\dfrac{\tilde{\lambda}\left(\tilde{\beta}+\tilde{\gamma}\right)}{\tilde{\mu}\,\tilde{\gamma}} < 1$

If the system does not suffer breakdown ie, if letting $\tilde{\beta} = 0$ in (34) and (32) we have

$$P(Z) = \frac{(Z-1)\,\tilde{\mu}\left(1-\dfrac{\tilde{\lambda}}{\tilde{\mu}}\right)}{\left(\tilde{\lambda}+\tilde{\mu}\right)Z-\tilde{\lambda}Z^2-\tilde{\mu}} \quad \text{... (35)}$$

$$W_0 = \frac{\tilde{\mu}-\tilde{\lambda}}{\tilde{\mu}} = 1-\rho \quad \text{... (36)}$$

Now if L and L_q denote the steady state average system size and the average queue size then from (34)

$$L = P'(Z) = 1 = \frac{\tilde{\lambda}\left[\left(\dfrac{\tilde{\beta}\,\tilde{\mu}}{\tilde{\beta}+\tilde{\gamma}}\right)+\left(\tilde{\beta}+\tilde{\gamma}\right)\right]}{\tilde{\mu}\tilde{\gamma}-\tilde{\lambda}\left(\tilde{\beta}+\tilde{\gamma}\right)}$$

Using Little's Formula

$L_q = L - \rho$

We can also find the average time spend in the system W and the average time spent in the que W_q using Little's Formula

$$W = \frac{L}{\tilde{\lambda}} \quad \text{and} \quad W_q = \frac{L_q}{\tilde{\lambda}}$$

NUMERICAL EXAMPLE

A numerical example is done using function principle by taking

x = (0.01, 0.02, 0.03, 0.04)
y = (2.1, 2.2, 2.3, 2.4)
p = (1.1, 1.2, 1.3, 1.4)
q = (0.1, 0.2, 0.3, 0.4)

Trapezoidal fuzzy number operations :

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers then the arithmetic operations on $\tilde{A}$ and $\tilde{B}$ are given as

(i) Addition $\tilde{A} \oplus \tilde{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4]$

(ii) Subtraction $\tilde{A} \ominus \tilde{B} = [a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1]$

(iii) Multiplication $\tilde{A} \otimes \tilde{B}$ = $[a_1b_1, a_2b_2, a_3b_3, a_4b_4]$

(iii) Division $\tilde{A} \oslash \tilde{B}$ = $[a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1]$

Using function principle

$(p + q)^2 = (1.44, 1.96, 2.56, 3.24)$

We can now calculate the fuzzy average system size

$$L = \frac{x\,[py + (p + q)^2]}{(p + q)[qy - x(p + q)]}$$

= (0.0419, 0.0868, 0.3033, 1.4285)

The fuzzy average time spent in the system is

$$W = \frac{L}{x} = (0.0475, 2.8933, 15.165, 142.85)$$

Utilization Factor

$$\tilde{\rho} = \frac{x}{y} = (0.0041667, 0.008695, 0.013637, 0.019048)$$

We can now calculate the fuzzy probability that there are no customers in the system and the service channel is in the operating state

$W_0 = 1 - \tilde{\rho}$ = (0.98095, 0.986363, 0.991305, 0.99583)

We can calculate the fuzzy average queue size L_q and fuzzy average time spent in queue W_q can be calculated using the Littles Formula.

$L_q = L - \tilde{\rho}$ = (0.02285, 0.073163, 0.294605, 1.42433)

& $W_q = \frac{L_q}{x}$ = (0.57125, 2.43876, 14.73025, 142.433)

CONCLUSION

An of $FM^x / FM^{(a,b)} / 1$ queue with server breakdowns have been studied. The PGF of queue size at an arbitrary time epoch is obtained. Some performance measures are also derived. Some particular cases are also discussed. An example is also given.

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