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Review Article

APPROXIMATE ASSESSMENT AND ANALYSIS OF FREE VIBRATION IN COUPLING SEISMIC WAVES AND VIBRATION BETWEEN STRUCTURE AND SOIL

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ABSTRACT
As seismic and wind-induced loads are the most important factor for difference between short to medium and high structure, so recognition of vibrations emitted from high structuregains importance because of seismic waves. If the structure fluctuates and speeds up; people's tranquility is taken away. In this research, a mathematical model has been offered to determine frequency and shape function of high structure with central core, belt truss and outrigger. Frequency values and shape functions of vibrational mode of structure is obtained after extracting nonlinear equations of correlation by using separation method compare to place and space and employing boundary conditions. To correctly examine equations obtained by 5 floors-steel structure modeled by PERFORM 3D and its result has been compared by method results.

Keywords: *High Structure, Free Vibrations, Hamilton Principle of Change, Nonlinear Analysis, Seismic Waves*

INTRODUCTION
Free vibration analysis plays a highly important role in designing high structures especially the first mode. Shape of first mode in vibration inducted by earthquake is dominant elements of high structure. Therefore, it is important to calculate natural frequency and mode shapes based on high structures. In recent years, many researches have been given to civil engineering to obtain free vibration for high structures. As it previously stated, many research has been conducted by different investigators on free vibrations of high structures where they have modeled free vibrations of console beam. Different researchers have considered various parameters in their methods. Free vibration analysis plays an essential role in designing high structures, especially first mode of vibration, as this mode is dominant mode in vibrational response of high structure loaded by wind or earthquake (Skattum, 1974).
Wang obtained a direct formula of forth-level Sturm-Liouville differential Equations to calculate natural frequency of tube in tube high buildings. The main variable is selected by Fourth-degree Sturm-Liouville differential Equations and in accordance with end point conditions. Wang continued his work to correct ODE solution plan and calculates particular values from numerical solution for free vibration of tube in tube high structure. An effective attitude is based on classic power series method for solving conventional differential issues (Basu, 1983; Basu, 1984). In recent years, Eisen Berger offered solution of power series for obtaining beams vibration frequency with one or two variable elastic parameters.
A method has been offered for analyzing beam vibration with variable hardness parameters. Shearing transformation is just considered for framed-shaped structures in the above method. Beam vibrations with variable cross section were analyzed and some functions were also introduced for mass distribution and stiffness along the beam. Local differential vibration equation is offered a kind of bracing system as a number of graphs those results for such design is highly complex. Design table has been offered for structural vibration frequency with low length to width relation.
Design table for a number of vibration frequencies of bracing systems has been offered which is usable result for narrow frame and shearing wall systems (Rosman, 1973; Rosman, 1974). Approximate expression has been offered for vibratory frequency of frame structures, which its accuracy is between -16% and +30%. A method for calculating vibration of one or more bracing system has been offered

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where offered solutions to especial range of limited parameters. Some formula is introduced for calculating radioactive frequency that is contains general application, formula error is between 0% and -16% in the first vibration frequency. So method precision is increased considering the property of column compression capability (Li, 1994; Li, 1999).

Recently a new method has been introduced for estimating vibratory frequency of high structure in which alternative prismatic cantilever beam contains three kinds of flexural, shearing and stiffness.

Power series solution for computing vibratory frequency of a beam has been applied. Power series method has been provided for computing vibratory frequency of a flexible device with moving mass. In this research, high structures with combined system is modeled as modeled mathematical model and by the use constant beam method and Hamilton principle of change is affected by coupling seismic waves (pair of seismic acceleration) is obtained.

Analysis Method

System of circumference frame with core; belt truss and outrigger are formed by near column and in structural medium; this columns are connected by deep beam in every flood to perimeter column, and perimeter frame is connected to core by outrigger (Figure 1). In this research, the effect of belt truss and outrigger is considered as a concentrated twisting spring (Figure 2).

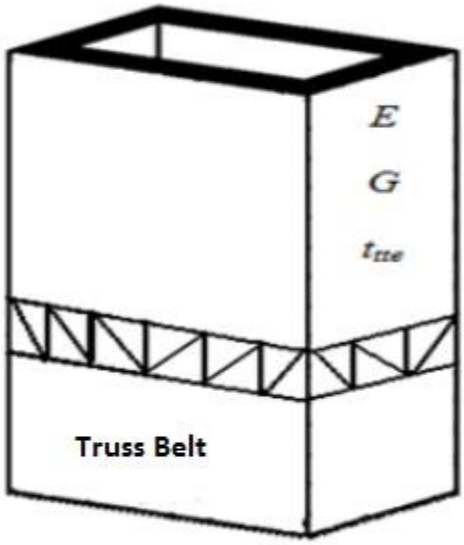

Figure 1: Belt Truss

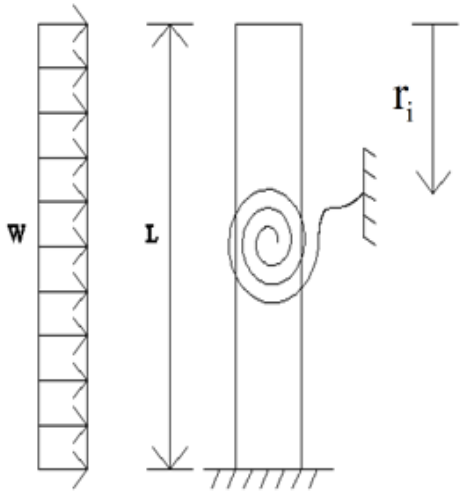

Figure 2: Effect of Belt Truss

According to Hook's law, axial strain is:

$$\varepsilon(z,t)=\frac{\partial u(z,t)}{\partial z} \quad (1)$$

And also tension and strain relation is:

$$\sigma(z,t)=E\varepsilon(z,t) \quad (2)$$

Considering above relation, axial force for the studied system is:

$$N(z,t)=\sigma(z,t)A(z)=\varepsilon(z,t)EA(z)=\frac{\partial u(z,t)}{\partial z}EA(z) \quad (3)$$

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Kinetic energy for this system is:

$$T(z,t)=\frac{1}{2}\int_{\circ}^{L}\bar{m}\left(\dot{u}(z,t)\right)^{2}dz+\frac{1}{2}M\left(\dot{u}(r,t)\right)^{2} \tag{4}$$

Also, potential energy is equal with following values:

$$V(z,t)=\frac{1}{2}\left[\int_{\circ}^{L}EI\left(u''(z,t)\right)^{2}+N\left(u'(z,t)\right)^{2}+AG\left(u'(z,t)\right)^{2}dz\right]+$$
$$\frac{1}{2}\sum_{i=1}^{n}K_{i}\left(u'(r_{i},t)\right)^{2}+\frac{1}{2}\sum_{i=1}^{n}k_{iN}\left(u(r_{i},t)\right)^{2} \tag{5}$$

So that in the system, flexural stiffness with El(z), shear stiffness with GA(z), axial stiffness with EA(z) and structure height unit mass with m(z), dynamic displacement with u (z, t), structure total height with L, belt truss torque stiffness and outrigger with k and axial stiffness of outrigger with Kn in which N=1:n, denotes the number of belt truss in the structure, r denotes the position of outrigger and belt truss in the structure height.

Considering the Hamilton principle, form of energy of energy function is express as follows:

$$L=L\left(t,u,\dot{u},u'\right) \tag{6}$$

Similarly, energy variation is:

$$\delta A=\int_{t_1}^{t_2}\int_{0}^{L}L\left(t,u,\dot{u},u'\right)dzdt\Rightarrow\int_{t_1}^{t_2}\int_{0}^{L}\left(\frac{\partial L}{\partial u}\delta u+\frac{\partial L}{\partial\dot{u}}\delta\dot{u}+\frac{\partial L}{\partial u'}\delta u'\right)dzdt \tag{7}$$

With bound conditions are as follows:

$$t_1<t<t_2\,,\ 0<z<L \tag{8}$$

Considering the Hamilton principle, general energy of system is:

$$L=T-V \tag{9}$$

Dynamic function A is expressed as structure overall energy integral between tow range of t1 and t2:

$$A=\int_{t_1}^{t_2}Ldt=\int_{t_1}^{t_2}\left(T-V\right)dt \tag{10}$$

Considering the Hamilton principle for structure energy is static, then $A\delta=0$, therefore for applying variations operator on the equation (10), we have:

$$\delta A=\int_{t_1}^{t_2}\left(\delta T-\delta V\right)dt \tag{11}$$

By substituting equation (4) and 95) in (110, we have:

$$\delta A=\int_{t_1}^{t_2}\int_{0}^{L}\left(\bar{m}\dot{u}(z,t)\delta\dot{u}-EIu''(z,t)\delta u''-Nu'(z,t)\delta u'-GA\,u'(z,t)\delta u'\right)dz\,dt+$$
$$\int_{t_1}^{t_2}M\dot{u}(r,t)\delta\dot{u}\,dt-\int_{t_1}^{t_2}[\sum_{i=1}^{n}K_{i}u'(r_{i},t)\delta u'+\sum_{i=1}^{n}k_{iN}\,u(r_{i},t)\delta u]dt \tag{12}$$

Doing integral by part from eq with respect to place and time (12), we have:

$$\delta A=\int_{t_1}^{t_2}\int_{0}^{L}\left(m\ddot{u}(z,t)+EIu''''(z,t)-GAu''(z,t)-Nu''(z,t)\right)\delta u dzdt-\int_{0}^{L}m\dot{u}(r,t)\,\delta u\Big|_{t_1}^{t_2}dz-$$
$$\int_{t_1}^{t_2}\left(M\dot{u}(r,t)\delta\dot{u}\right)dt+\int_{t_1}^{t_2}\left(-EIu'''(z,t)+GAu'(z,t)+Nu'(z,t)+\sum_{i=1}^{n}k_{iN}u(r_{i},t)\right)\delta u\Big|_{0}^{L}dt-$$
$$\int_{t_1}^{t_2}\left(\left[EIu''(z,t)+\sum_{i=1}^{n}K_{i}u'(r_{i},t)\right]\delta u'\right)dt \tag{13}$$

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Assuming that height unit mass, flexural stiffness, axial stiffness and shear stiffness do not vary in outrigger and height of structure, structure vibration equation is expressed as follows:

$$\begin{aligned}
&u(z,t)=0 && z=0\\
&u'(z,t)=0 && z=0\\
&-EIu'''(z,t)+GAu'(z,t)+Nu'(z,t)+\sum_{i=1}^{n}k_{iN}u(r_i,t)=0 && z=L\\
&EIu'''(z,t)+\sum_{i=1}^{n}k_i u'(r_i,t)=0 && z=L\\
&m\ddot{u}(z,t)+EIu''''(z,t)-GAu''(z,t)-Nu''(z,t)=0 && 0\le z\le L
\end{aligned} \tag{14}$$

Within above equations prime and dot denote derivatives with respect to place and time, respectively. Solving above equations take place through variables separation, in which it is assumed that the answer can be shown as product of place function and time function:

$$u(z,t)=\phi(z)\,y(t) \tag{15}$$

Where in the above equation, $\Phi(Z)$ is the form function of shear mode in the structure height and y(t) is harmonic function with vibratory frequency ω. By substituting equation (15) in equation (14):

$$\frac{\phi^{iv}(z)}{\phi(z)}-\frac{GA}{EI}\frac{\phi''(z)}{\phi(z)}-\frac{N}{EI}\frac{\phi''(z)}{\phi(z)}+\frac{\ddot{y}(t)}{y(t)}\frac{m}{EI}=0 \tag{16}$$

In the following equation, there is only x function at the left side of the equation and t function at the right side. For establishing the equality of two sides of the equation as well as the equation would have answer, it is necessary to take the both side of the equation as a constant value, so we have:

$$\frac{\phi^{iv}(z)}{\phi(z)}-\frac{GA}{EI}\frac{\phi''(z)}{\phi(z)}-\frac{N}{EI}\frac{\phi''(z)}{\phi(z)}=-\frac{\ddot{y}(t)}{y(t)}\frac{m}{EI}=a^4 \tag{17}$$

That after separating equation (17):

$$\ddot{y}(t)+\frac{EIa^4}{m}y(t)=0 \tag{18}$$

For simplifying equation (18):

$$\omega^2=\frac{a^4EI}{m} \tag{19}$$

That after substituting equation (19) in equation (18) we have:

$$\ddot{y}(t)+\omega^2 y(t)=0 \tag{20}$$

Equation (20) is a second order differential equation with constant coefficient, and the answer of this equation is as follows:

$$y(t)=A\cos\omega t+B\sin\omega t \tag{21}$$

The above equation shows the structure vibration with frequency ω.
For simplifying equation (17), dimensionless parameters are defined as follows:

$$g^2=\frac{GA}{EI} \qquad r^2=\frac{N}{EI} \tag{22}$$

In addition to right side of equation (17), left side of this equation should be equal to constant value a^2:

$$\frac{\phi^{iv}(z)}{\phi(z)}-g^2\frac{\phi''(z)}{\phi(z)}-r^2\frac{\phi''(z)}{\phi(z)}=a^4 \tag{23}$$

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For simplifying equations, defined dimensionless parameters are substituted in the equation (17), so we have:

$$\phi^{iv}(z)-(g^2+r^2)\phi''(z)-a^4\phi(z)=0 \tag{24}$$

For solving equation (24), assuming that $\Phi(Z)$ vibratory mode form function in the structure height is:

$$\phi(z)=Ge^{jz} \tag{25}$$

By substituting equation (25) in (24), we have:

$$Ge^{jz}(j^4-j^2g^2-r^2j^2-a^4)=0 \tag{26}$$

Therefore roots of equation (26) are obtained as follows:

$$j_{1,2}=\pm i\alpha \qquad j_{3,4}=\pm\beta \tag{27}$$

Where, α and β are respectively:
After substituting each one of these roots in the equation (25), an answer for equation (24) is yielded as follows:

$$\phi(z)=G_1e^{i\alpha}+G_2e^{-i\alpha}+G_3e^{\beta}+G_4e^{-\beta} \tag{28}$$

Exponential function in above relation can be expressed as trigonometric functions. Therefore, equation (29) are expressed as:

$$\phi(z)=A_1\cos\alpha z+B\sin\alpha z+C\cosh\beta z+D\sinh\beta z \tag{29}$$

By applying boundary condition on the equation (14) and substituting it in equation (30), these equations are obtained:

$$\phi(0)=A_1+C \tag{30}$$

$$\begin{aligned}
&\phi'(0)=B\alpha+D\beta\\
&\phi'(L)=-A_1\alpha\sin\alpha L+B\alpha\cos\alpha L+C\beta\sinh\beta L+D\beta\cosh\beta L\\
&\phi''(L)=-A_1\alpha^2\cos\alpha L-B\alpha^2\sin\alpha L+C\beta^2\cosh\beta L+D\beta^2\sinh\beta L\\
&\phi'''(L)=A_1\alpha^3\sin\alpha L-B\alpha^3\cos\alpha L+C\beta^3\sinh\beta L+D\beta^3\cosh\beta L
\end{aligned} \tag{31}$$

$$\begin{cases}
\phi(z)_{z=0}=0\\
\phi'(z)_{z=0}=0\\
\phi''(z)_{z=L}+\sum_{i=1}^{n}K_i\phi'(z)_{z=r_i}=0\\
\phi'''(z)_{z=L}+g^2\phi'(z)_{z=L}+r^2\phi'(z)_{z=L}+\sum_{i=1}^{n}k_{iN}\phi(z)_{z=r_i}=0
\end{cases}$$

For simplifying the equation we have:

$$c_1=\sum_{i=1}^{n}K_i \qquad c_2=\sum_{i=1}^{n}k_{iN} \qquad c_3=g^2+r^2 \tag{32}$$

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For obtaining a non-trivial solution for above relation, the determinants of the coefficient should be zero:

$$\begin{Bmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha & 0 & \beta \\ -\alpha^2\cos\alpha L & -\alpha^2\sin\alpha L & \beta^2\cosh\beta L & \beta^2\sinh\beta L \\ -\sum_{i=1}^{n} K_i\alpha\sin\alpha b & +\sum_{i=1}^{n} K_i\alpha\cos\alpha b & +\sum_{i=1}^{n} K_i\beta\sinh\beta b & +\sum_{i=1}^{n} K_i\beta\cosh\beta b \\ \alpha^3\sin\alpha L+\sum_{i=1}^{n} K_{iN}\cos\alpha r_i & \alpha^3\cos\alpha L+\sum_{i=1}^{n} K_{iN}\sin\alpha r_i & \beta^3\sinh\beta L+\sum_{i=1}^{n} K_{iN}\cosh\beta r_i & \beta^3\cosh\beta L+\sum_{i=1}^{n} K_{iN}\sinh\beta r_i \\ -(g^2+r^2)\alpha\sin\alpha L & +(g^2+r^2)\alpha\cos\alpha L & +(g^2+r^2)\beta\sinh\beta L & +(g^2+r^2)\beta\cosh\beta L \end{Bmatrix} \begin{bmatrix} A_1 \\ B \\ C \\ D \end{bmatrix} = 0 \quad (33)$$

After putting the determinant of the coefficient equal to zero, by hybrid vibratory structural feature equation can be obtained that is the first hybrid structural natural frequency.

$$\begin{aligned} &\alpha\beta^5\cosh(\beta L)^2-\alpha^5\beta\cos(\alpha L)^2+\alpha^5\beta\sin(\alpha L)^2-\alpha\beta^5\sinh(\beta L)^2-c_3\alpha^3\beta\cos(\alpha L)^2+c_3\alpha\beta^3\cosh(\beta L)^2 \\ &-c_3\alpha^3\beta\sin(\alpha L)^2-c_3\alpha\beta^3\sinh(\beta L)^2-c_2\beta^3\cosh(\beta L)\sin(\alpha r)+c_3\alpha^3\cos(\alpha L)\sinh(\beta r)-\alpha^2\beta^4\sin(\alpha L)\sinh(\beta L) \\ &+\alpha^4\beta^2\sin(\alpha L)\sinh(\beta L)-c_1\alpha^2\beta^3\cosh(\beta L)\sin(\alpha r)-c_1\alpha^3\beta^2\cos(\alpha L)\sinh(\beta r)-2c_3\alpha^2\beta^2\sin(\alpha L)\sinh(\beta r) \\ &-c_1c_2\alpha\beta\cos(\alpha r)^2-c_3c_1\alpha\beta\cos(\beta r)^2-c_2c_1\alpha\beta\sin(\alpha r)^2+c_1c_3\alpha\beta\sinh(\beta r)^2-c_3\alpha\beta^3\cos(\alpha L)\cosh(\beta L) \\ &+c_3\alpha^3\beta\cos(\alpha L)\cosh(\beta L)-c_1c_2\beta^2\sin(\alpha r)\sinh(\beta r)+c_1c_3\alpha^2\sin(\alpha r)\sinh(\beta r)-c_1\alpha^4\beta\cos(\alpha L)\sin(\alpha r) \\ &-c_1\alpha^4\beta\cos(\alpha r)\sin(\alpha L)-c_2\alpha^2\beta\cos(\alpha L)\sin(\alpha r)+c_2\alpha^2\beta\cos(\alpha r)\sin(\alpha L)+c_1\alpha\beta^4\cos(\alpha r)\sinh(\beta L) \\ &+c_1\alpha^4\beta\cosh(\beta r)\sin(\alpha L)+c_2\alpha\beta^2\cos(\alpha r)\sinh(\beta L)+c_1\alpha\beta^4\cosh(\beta L)\sinh(\beta r)-c_1\alpha\beta^4\cosh(\beta r)\sinh(\beta L) \\ &-c_3\alpha^2\beta\cosh(\beta r)\sin(\alpha L)+c_3\alpha\beta^4\cosh(\beta L)\sinh(\beta r)-c_3\alpha\beta^2\cosh(\beta r)\sin(\beta L)-c_1c_3\alpha^2\beta\cos(\alpha L)\sin(\alpha r) \\ &+c_1c_3\alpha^2\beta\cos(\alpha r)\sin(\alpha L)-c_1c_3\alpha\beta^2\cos(\alpha L)\sinh(\beta r)+c_1c_3\alpha\beta^2\cos(\alpha r)\sinh(\beta L)+c_1c_3\alpha^2\beta\cosh(\beta L)\sin(\alpha r) \\ &-c_1c_3\alpha^2\beta\cosh(\beta r)\sin(\alpha L)+c_1c_3\alpha\beta^2\cosh(\beta L)\sinh(\beta r)-c_1c_3\alpha\beta^2\cosh(\beta r)\sinh(\beta L)+c_1c_2\alpha\beta\cos(\alpha r)\cosh(\beta r) \\ &c_1c_3\alpha\beta\cos(\alpha r)\cosh(\beta r)=0 \end{aligned} \quad (34)$$

And coefficient A1, B, C and D are related to each other in following manner:

$C = -A_1$

$$D = -\frac{\alpha}{\beta}B,$$

$$A_1 = B\left(\frac{\frac{\alpha}{\beta}\left(\beta^2\sinh\beta L+c_1\beta\cosh\beta b\right)-\alpha^2\sin\alpha L+c_1\alpha\cos\alpha b}{-\alpha^2\cos\alpha L-c_1\alpha\sin\alpha b+\alpha^2\sin\alpha L-c_1\alpha\cos\alpha b}\right)$$

$$\lambda = \frac{\frac{\alpha}{\beta}\left(\beta^2\sinh\beta L+c_1\beta\cosh\beta b\right)-\alpha^2\sin\alpha L+c_1\alpha\cos\alpha b}{-\alpha^2\cos\alpha L-c_1\alpha\sin\alpha b+\alpha^2\sin\alpha L-c_1\alpha\cos\alpha b}$$

$$A_1 = \lambda B$$

(35)After identifying coefficients, equation (30) can be written as follows:

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$$\phi(x) = \lambda B\cos\alpha x + B\sin\alpha x - \lambda B\cosh\beta x - \frac{\alpha}{\beta}B\sinh\beta x \quad (36)$$

Numerical Example

In this paper, for examining high structure vibrations of a 50 stories steel building with hybrid system is modeled in PERFOEM3D software under coupling seismogram. Dimensions of all beams and columns are 0.8x0.8 m^2, distance of columns is 2.5 m and height of each story is 3m.
Obtained results are shown in table (1).

Number of Belt Truss and Outrigger	Natural Frequency		Error Percentage
	Theoretical Method	SoftwarePERFORM3D	
1	1.9831	2.3015	13.8
2	2.4361	2.5103	2.95
3	2.4806	2.5700	3.4
4	2.5363	2.5942	2.2
5	2.5681	2.6235	2.1

Conclusion

Obtained results from this study show that using this method gives rise to rapid access to frequency values and vibratory form function and similarly using outrigger and belt truss system in high structures leads to increase structure lateral stiffness and this result in increasing structure natural values.
Following results can be concluded from current paper:

1- An approximate solution process for free vibration analysis under earthquake seismogram couple is presented for high structures.
2- Proposed method in this paper can calculate natural frequencies rapidly and accurately by calculator and without using complicated computer programs.
3- Impacts of parameters such as flexural rigidity, length unit mass and total height of structure for natural frequency are discussed in high buildings.
4- One can obtain an appropriate accuracy of calculating natural frequency of structures by increase of number of outrigger and belt truss.
5- Proposed method in this paper can be used as an alternative for evaluating natural frequencies of free vibration of hybrid structures (outrigger and belt truss) in the preliminary stage of building design.

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