PROPOSITION OF A MATHEMATICAL MODEL FOR INTEGRATION OF MACHINERY AND SCHEDULING OF CELLULAR MANUFACTURING SYSTEM THROUGH GENETIC ALGORITHM

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ABSTRACT
There are three key issues involved in the design of a cellular manufacturing system (CMS), cell configuration (CF), group layout (GL) and group scheduling (GS), which should be considered for achieving successful implementation of cellular manufacturing and simultaneously finding an optimal solution. Therefore, this paper attempted to propose a mixed integer nonlinear mathematical model (MINLP) so as to integrate these three issues in a cellular manufacturing system. The unique characteristic applied in the model is the unification of several important design parameters such as operation sequence, operation time, shifting time, intracellular and extracellular group layout. The objective function of this model consists of costs and three parts including minimization of total completion time, delay fees for each segment and intracellular and extracellular shifts. Moreover, the GA is proposed to solve the problem due to NP-hard. Afterwards, the performance of the model was verified through a numerical example encoded through LINGO SYSTEM where the computational results were presented. The results showed that the cell configuration and machine layout in cells and the manufacturing space and manufacturing scheduling led to higher productivity through reduction of costs incurred by piece shifts, system maintenance (which depends on total operation time) and delay fees in delivery of pieces. One of the advantages of this model is that it completes all the steps together in one go.

Keywords: Cell Manufacturing System, Cell Configuration, Group Layout, Group Scheduling, Genetic Algorithm

INTRODUCTION
Today, in modern industrial competition, every company needs to be capable of showing quick reaction to sudden changes in the market demand for its products. The increased flexibility and efficiency have become integral parts of the main objectives of every manufacturing company. One of the key techniques to achieve this goal involves cell manufacturing, which is one of the important methods of group technology (GT). That is generally discussed in modern production systems, including flexible manufacturing systems and Just-In-Time (JIT). The Group Technology is a manufacturing technique in which pieces with similar characteristics are placed in one group and a set of machines used for their manufacturing are properly classified and deployed within the same unit (Allahverdi, 2006). Cell configuration is done in such a way that the least movement of parts (materials) take place, directing the movement as much as possible in the manufacturing line of a cell, hence higher shift can increase costs, time and consumption of resources through the manufacturing process. At the same time with deciding on the configuration, the scheduling and planning should be considered and decided. That is because time is always a limitation, i.e. a competitive issue and even the survival key in a competitive top quality market, which leads to higher efficiency and optimum use of available resources and ultimately increasing profitability. The main goals in scheduling are to avoid delay in delivery to the customer and reduce total manufacturing time of products. Hence, decision should be adopted regarding the best sequence of operations and scheduling of activities together with configuration and layout of cells (Filho, 2006).

Problem Statement
Cell manufacturing is one of the major applications of Group Technology. The designing of an efficient cell manufacturing system entails three basic steps of forming a group layout and group scheduling.
Research Article

Despite the impact on each of the three steps on one another, most studies in this field have dealt with these steps separately. This research involves an integrated model of CMS, composed of cell configuration, intracellular and extracellular layout and manufacture scheduling. The cell configuration focuses on allocating a group of machines to the cells, categorization of the family parts and allocation of family pieces to form independent cells. Along with this grouping, the machines are deployed through insertion of continuous variables into cell respective positions. Moreover, the manufacture scheduling is done at the same time with above steps. In the case of cell configuration, \( m \) cell systems are categorized in \( c \) cells in a way that \( p \) pieces are manufactured as much as possible in one cell. If there are intracellular shifts, they need to be minimized. On the issue of intracellular layout, the machines are aligned within the cells in a way that pieces experience minimal intracellular shift, since there will be lower costs and time. On the issue of extracellular layout, the cells are arranged in a production environment so that the parts experience minimal shift, since there will be lower costs and time (Balakrishnan, 2005).

On the issue of the timing of the cell manufacturing system, it is assumed to be \( p \) machine stationed in \( c \) cells generate segment. When a piece is being processed on a machine, no other piece can be processed on the machine. The piece should be first removed from the machine or it needs to wait for the process to be finished on the previous pieces. The problem does not allow any breakdown in the processing flow. Therefore, the priority in processing occurs on parts, where decision is made as to what piece to be processed ahead of others, i.e. manufacture scheduling. In scheduling, the processing order of components is determined on the machines. Furthermore, machines are selected to attain the goals in case there are parallel machines for processing the operations. The objectives in scheduling are to minimize the total duration of manufacture and minimization of delay in delivery of individual parts considering their deadlines. The configuration procedure and layout of cells and intracellular processing and selection of a process from parallel machines at time of manufacture can as affect total manufacturing time and individual parts. Hence, all the four steps above, according to the goals, occur simultaneously defining the problem below:

Configuration, layout of machines (intracellular and extracellular) and cells in a continuous and group scheduling serve to minimize the total cost of intra-extracellular shift, total duration and delay fees (Ahi, 2009).

Literature

GA was first proposed by John Holland at the University of Michigan. The evolutionary strategies and programs were developed as evolutionary computation methods by Rechenberg, Schwefe, Fogogel and Koza.

The optimization methods inspired by nature significantly differ from conventional methods of optimization. In conventional methods, each solution to a new candidate is selected when it improves the objective function value, while all the solutions to a new candidate are granted the chance to be selected in algorithms inspired by nature. Genetic algorithm is one of the most important metaheuristic algorithms is used to optimize different functions. In this algorithm, the past information is extracted based on inheritance and adopted throughout the search process.

The basic concept of a genetic algorithm was initially presented by Holland followed by Goldberg (1989) who provided description. Genetic algorithms are stochastic search techniques functioning based on natural selection and natural geneology. These algorithms are fundamentally different from traditional search and optimization methods summarized by Goldberg as follows (Guyton, 2000):

1. Genetic algorithm deals with a set of coded solutions, not the solutions per se.
2. GA adopts the fitness function information, rather than derivatives or other auxiliary sciences.
3. GA uses stochastic transition rules, not absolute rules.

An article titled "a mathematical model for the cell manufacturing layout", a two-stage meta-heuristic algorithm and genetic algorithm approach was proposed for solving the cell layout problem and cell
layout was discussed. The results demonstrated that the machine cell and family piece at the first state are identified through mathematical model. Step 2 involves the macro approach so as to examine the problem of cell layout according to the sequence of operations that minimizes movement between cells. In stage 2, the cell layout is formulated according to linear layout as a second-degree allocation problem. The two mathematical models are NP-Hard adopted to solve the genetic algorithms (Defersha, 2006).

In an article titled "optimization of genetic algorithms related to the cargo industry", a genetic algorithm was developed for a set of manufacturing cells, or all production facilities. The model was tested through the large-scale data provided by a capital cargo company. The results of this study showed that redesigning of facilities first determine the intracellular layout and then locates the cells between empty departments. The budget restriction imposed extra implementation of additional cells limited to investing in any financial period (Hicks, 2006).

In an article titled "manufacturing and designing of cell manufacturing by genetic algorithm", a hierarchical genetic algorithm was developed so as to simultaneously specify the manufacturing cells and group layout. Two important systems for cell design proposed by the hierarchical structure of chromosomes were coded. Moreover, a new selective profile and a mutation operator group were introduced (Xiaodan, 2007).

In fact, the production systems need to be modified in a way to be able to manufacture products through minimum cost, highest quality and shortest possible duration for timely delivery to customers. Furthermore, the systems should be able to as quickly adapt themselves to changes in demand and the design of products without the need to reinvest (Safai, 2008).

In an article titled "solving a multi-objective scheduling problem in cellular manufacturing system using hybrid algorithm", a new model was presented for group scheduling of manufacturing cells obtained through intracellular scheduling of pieces in cells and in cells intracellular sequences. Then, the model was solved through a meta-heuristic known as ScatterSearch (Tavakoli, 2010).

New and Innovative Aspects of Research

This research intended to adopt an integrated approach unlike previous studied so as to simultaneously the problem of group scheduling and cell configuration and machine layout (intracellular and extracellular) in a cell manufacturing system within a continuous space aimed at minimizing the total cost of replacement and intracellular movement, the total manufacture time and delay fees. Hence, this integrated approach can be regarded as the innovation in this study in terms of quantity using mathematical modeling. Knowing the type of modeling, accurate or meta-heuristic methods appropriate in this area can be great help to evaluate performance level, which in turn proposes another innovation.

The synchronization of cell configuration steps, intracellular and extracellular layout and manufacture scheduling given the above factors and the continuity of the manufacturing system are the distinctive features of this research.

Nonlinear Mathematical Model

\[ \text{Min} = Q \times C_{\text{max}} (1 - 1 - 3) \]

\[ + \sum_{i=1}^{p} \left( P_i y_i \times \max \{0, CTP_i - dd_i \} \right) (2 - 1 - 3) \]

\[ + \sum_{k=2}^{K_p} \sum_{i=1}^{P} \sum_{j=1}^{M} \sum_{c=1}^{C} \left( \sum_{k'=1}^{K_m} Z_{k'j}^{k_i} \right) \times \left( \sum_{k''=1}^{K_m} Z_{k''j}^{k'_i} \right) \times \left[ (V_j \times V_{j'} \times CI_i) + (V_{j'} \times (1 - V_{j'}) \times CO_i) \right] \]

\[ \times D_{jj'} (3 - 1 - 3) \]

Subject to
\[
\begin{align*}
\sum_{c=1}^{M} V_{jc} &= 1 \quad \forall j \quad (2 - 3) \\
\sum_{j=1}^{M} V_{jc} &\leq UMC \quad \forall c \quad (3 - 3) \\
\sum_{j=1}^{M} V_{jc} &\geq LMC \quad \forall c \quad (4 - 3) \\
\sum_{k'=1}^{K_m} \sum_{j=1}^{M} Z_{k_i}^{j'} &= 1 \quad \forall k, i \quad (5 - 3) \\
\sum_{j=1}^{M} (\sum_{k'=1}^{K_m} Z_{k_i}^{j'}) \times T_{kij} &> 0 \quad \forall k, i \quad (6 - 3) \\
CTM_{ij} &= \sum_{i=1}^{P} \left( \sum_{k=2}^{K_p} Z_{k_i}^{j'} \times \left[ T_{kij} + \sum_{k'=1}^{K_m} \sum_{j'=1}^{M} Z_{k_i}^{j'} \left( CTM_{k'-j_i} + (D_{jj'} \times TT_i) \right) \right] \right. \\
&\quad \left. + Z_{T_i}^{j'} \times T_{1ij} \right) \quad \forall j \quad (7 - 3) \\
CTM_{k' j} &= \sum_{i=1}^{P} \left( \sum_{k=2}^{K_p} Z_{k_i}^{j'} \times \left[ T_{kij} \\
&\quad + \max \left\{ CTM_{k'-1 j_i}, \sum_{k'=1}^{K_m} \sum_{j'=1}^{M} Z_{k_i}^{j'} \left( CTM_{k'-j_i} + (D_{jj'} \times TT_i) \right) \right] \right. \right. \\
&\quad \left. \left. \times \left[ T_{1ij} + CTM_{k'-1 j_i} \right] \right) \quad \forall k' > 1, j \quad (8 - 3) \\
CTP_i &= \sum_{k'=1}^{K_m} \sum_{j=1}^{M} Z_{k_p i}^{j'} \cdot CTM_{k' j} \quad \forall i \quad (9 - 3) \\
C_{\text{max}} &= \max \{ \forall i; CTP_i \} \quad (10 - 3) \\
\alpha_j &= \frac{1}{2} (x_j + x''_j) \quad \forall j \quad (11 - 3) \\
\beta_j &= \frac{1}{2} (y_j + y''_j) \quad \forall j \quad (12 - 3) \\
D_{jj'} &= \left| \alpha_j - \alpha_{jj'} \right| + \left| \beta_j - \beta_{jj'} \right| \quad j \neq j' \quad (13 - 3) \\
\begin{cases} x''_j - x'_j &= L_j \\
y''_j - y'_j &= W_j \end{cases} \quad (14 - 3) \\
\begin{cases} x''_j - x'_j &= L_j \\
y''_j - y'_j &= W_j \end{cases} \quad (15 - 3)
\end{align*}
\]
Objective Function

The objective function includes the cost of maintenance and costs related to time, delay fees and two types of shift costs. Equation (3.1.1) is related to system maintenance costs and time costs which are obtained through multiplication of total operation time by system maintenance costs per unit of time. This cost includes the depreciation and maintenance. This equation reduces the time of task completion. Equation (3.1.2) involves the total delay fee of pieces. Equation (3.1.3) is related to the intracellular and intercellular shift. The intracellular cost occurs when two consecutive operations are performed on one piece within a cell, while extracellular cost occurs when two consecutive operations of one piece are performed within two cells. The product indicates whether two consecutive operations of piece \( i \) performed on two machines \((j, j')\) are located in cell \( c \) or not. Similarly, if the product is equal to zero means, the \( k \)th operation on piece \( i \) will be performed on a machine located on cell \( c \) as one of its sequences, while operation \( k-l \) of the same machine will take place in another cell. Since the cost of intracellular shift is extremely high, machines are arranged in cells in a way to reduce as much as possible the intracellular shift.

Even if the pieces need to be shifted between two cells, the model will attempt to move those machines with shortest distance from one another. A piece may weigh more than another piece or be larger or more fragile, which make it difficult to shift. Thus, higher cost or time will be required for shifting. One feature of this model is that a travel time and a special handling cost for each piece has been considered based on the distance. Given that cost in this respect is directly correlated with the distance between the two machines, the model attempts to put the machines with higher rate of shift closer to one another.

Model Linearization
Because of non-linear terms in the model, the linearization methods were used to linearize the proposed model. Although most of the optimization software programs available are capable of solving nonlinear models, but experience has shown that it takes longer to solve non-linear models which mainly converge into a local optimal solution.

As a result, a global optimal solution was achieved through model linearization which was important in terms of the number of variables, limits, problem solving duration and the optimal solution with sample issues analyzed and compared.

The proposed model is nonlinear. There are non-linear terms in both objective function and the constraints where are linearized at this stage.

In terms (3.1.2), (3-8) and (3-10), the max is used to linearize through replacing each max term a variable and consider the constraints of that variable larger than individual components within the respective max. Since it is a matter of minimization, the mode will naturally juxtapose that variable with the max of those components. For example, the expression (3.1.2) has used max instead of variable $max\{0, CTP_i - dd_i\}$, where two constraints $\text{tr}ans_i \geq 0$, $\text{tr}ans_i \geq CTP_i - dd_i$ are added.

The expression (3-13) uses an absolute value, for linearization of which it suffices to define a "positive side" and a "negative side" to the absolute negative as follow:

$$\alpha_j - \alpha_j^+ = \alpha_{jj}^+ - \alpha_{jj}^- \beta_j - \beta_j^+ = \beta_{jj}^+ - \beta_{jj}^-$$

Then:

$$|\alpha_j - \alpha_j| = \alpha_{jj}^+ + \alpha_{jj}^-$$
$$|\beta_j - \beta_j| = \beta_{jj}^+ + \beta_{jj}^-$$

While $\alpha_{jj}^+$, $\alpha_{jj}^-$, $\beta_{jj}^+$, $\beta_{jj}^-$ are all non-negative variables.

In terms (3.1.3), (3-6), (3-7), (3-8), (3-9) and (3-10), a binary variable has been multiplied by another variable. Instead of multiplication of two variables for linearization in this case, another variable is defined and several new constraints are added. If both variables are multiplied are binary, the new variable will be binary too and two constraints are added. For example, each sigma within parentheses in expression (3.1.3) entails binary values. Instead of multiplication, the binary variable $ZZ_{klt_{i,j}}$ is used and the following constraints are added:

$$ZZ_{kij} \geq \sum_{k'} Z_{k'i}^{kj} + \sum_{k'} Z_{k'1i}^{kj} - 1$$
$$ZZ_{kij} \leq \sum_{k'} Z_{k'i}^{kj}$$
$$ZZ_{kij} \leq \sum_{k'} Z_{k'i}^{kj} - 1$$

Or:

$$2 ZZ_{kij} \leq \sum_{k'} Z_{k'i}^{kj} + \sum_{k'} Z_{k'1i}^{kj}$$
$$ZZ_{kij} \geq \sum_{k'} Z_{k'i}^{kj} + \sum_{k'} Z_{k'1i}^{kj} - 1$$
If one of the variables was not binary, the new variable will be defined similarly and two constraints will be added. For example, the expression (3.1.2) has used non-negative variable $ZCTM_{kik'ji}$ instead of $Z_{k-1,i}^{k'ji} \times CTM_{k'ji}$, where the following constraints are added.

\[
\begin{align*}
ZCTM_{kik'ji} & \geq CTM_{k'ji} - M(1 - Z_{k-1,i}^{k'ji}) \\
ZCTM_{kik'ji} & \leq CTM_{k'ji} \\
ZCTM_{kik'ji} & \leq MZ_{k-1,i}^{k'ji}
\end{align*}
\]

Where $M$ is a large number.

The variables used to linearize model can be seen in the following table.

<table>
<thead>
<tr>
<th>The converted variable in the linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$trdns_i$</td>
</tr>
<tr>
<td>$ZZ_{kijji}$</td>
</tr>
<tr>
<td>$VV_{jj'c}$</td>
</tr>
<tr>
<td>$ZZV_{kijji'c}$</td>
</tr>
<tr>
<td>$ZZVD_{kijji'c}$</td>
</tr>
<tr>
<td>$ZZVVD_{kijji'c}$</td>
</tr>
<tr>
<td>$ZCTM_{kik'ji}$</td>
</tr>
<tr>
<td>$ZD_{kik'ji}$</td>
</tr>
<tr>
<td>$ZCTM_{kik'ji}$</td>
</tr>
<tr>
<td>$ZD_{kik'ji}$</td>
</tr>
<tr>
<td>$ZCTM'_{kik'tj}$</td>
</tr>
<tr>
<td>$MX_{kik'tj}$</td>
</tr>
<tr>
<td>$ZMX_{kik'tj}$</td>
</tr>
<tr>
<td>$\alpha_{jj'}^+, \alpha_{jj'}^-$</td>
</tr>
<tr>
<td>$\beta_{jj'}^+, \beta_{jj'}^-$</td>
</tr>
</tbody>
</table>

**Linear Mathematical Model**

Linear model is as follows:

\[
\begin{align*}
\min P & = Q \times C_{\max} \\
& + \sum_{i=1}^{p} (Ply_i \times trdns_i) \\
\end{align*}
\]
\[ R = \sum_{k=1}^{K_p} \sum_{p=1}^{P} \sum_{j=1}^{J} \sum_{j'=1}^{J'} \sum_{c=1}^{C} \left[ \left( ZZVD_{kij} \times C_i \right) + \left( ZZVD_{kij'} - ZZVD_{kij} \times C_i \right) \right] \]  

Subject to

\[ \sum_{c=1}^{C} V_{jc} = 1 \quad \forall j \quad (31-3) \]

\[ \sum_{j=1}^{M} V_{jc} \leq UMC \forall c \quad (32-3) \]

\[ \sum_{j=1}^{M} V_{jc} \geq LMC \forall c \quad (33-3) \]

\[ \sum_{i=1}^{K_m} \sum_{j=1}^{M} Z_{kij}^{k'j} = 1 \quad \forall k, i \quad (34-3) \]

\[ \sum_{j=1}^{M} (\sum_{k=1}^{K_m} Z_{kij}^{k'j} \times T_{klj}) > 0 \quad \forall k, i \quad (35-3) \]

\[ CTM_{1ij} = \sum_{i=1}^{P} \left( \sum_{k=2}^{K_p} \left( ZZ_{k} \times T_{klj} \right) + \sum_{k=1}^{K_m} \sum_{j'=1}^{M} \left( ZZCTM_{kik'j'} + (ZZD_{kik'j'} \times TT_i) \right) \right) \]

\[ + Z_{11}^{ij} \times T_{1lj} \quad \forall j \quad (36-3) \]

\[ CTM_{k'i} = \sum_{i=1}^{P} \left( \sum_{k=2}^{K_p} \left( ZZ_{k} \times T_{klj} + ZMX_{kik'j'} + ZCTM_{1iklj} + (Z_{1i} \times T_{1lj}) \right) \right) \quad \forall k', i \quad (37-3) \]

\[ CTP_i = \sum_{k=1}^{K_m} \sum_{j=1}^{M} ZCTM_{kik'lj} \forall i \quad (38-3) \]

\[ C_{max} \geq CTP_i \forall i \quad (39-3) \]

\[ \alpha_j = \frac{1}{2} (x' + x'') \forall j \quad (40-3) \]

\[ \beta_j = \frac{1}{2} (y' + y'') \forall j \quad (41-3) \]

\[ D_{ij} = \alpha_{ij}^+ + \alpha_{ij}^- + \beta_{ij}^+ + \beta_{ij}^- \forall j \neq j' \quad (42-3:44-3) \]
Linearization constraints of the model:

\begin{align}
\text{Linearization constraints of the model:} \\
\end{align}

\begin{align}
&\alpha_j - \alpha_j' = a_j^+ - a_j^-; \beta_j - \beta_j' = \beta_j^+ - \beta_j^- \\
&\begin{cases}
x''_j - x'_j = L_j \\
y''_j - y'_j = W_j
\end{cases} \tag{45 – 3}
\end{align}

\begin{align}
&\begin{cases}
x''_j \leq UX_j + (1 - V_j)c_M \\
LX_j \leq x'_j + (1 - V_j)c_M \\
y''_j \leq UY_j + (1 - V_j)c_M \\
LY_j \leq y'_j + (1 - V_j)c_M
\end{cases} \tag{47 – 3:50 – 3}
\end{align}

\begin{align}
&\begin{cases}
x''_{j'} \leq x'_j + (1 - R_{jj'})M \\
x''_{j'} \leq x'_j + (1 - R_{jj'})M \\
y''_{j'} \leq y'_j + (1 - R_{jj'})M \\
y''_{j'} \leq y'_j + (1 - R_{jj'})M
\end{cases} \tag{52 – 3:55 – 3}
\end{align}

\begin{align}
&\begin{cases}
trns_i \geq 0 \\
\forall j, trns_i \geq CTP_i - dd_i \forall j \tag{56 – 3}
\end{cases}
\end{align}

\begin{align}
&MX_{kik'j} \geq CTM_{k' -1 , j} \\
&MX_{kik'j} \geq \sum_{k=1}^{M} \sum_{j=1}^{M} (ZCTM_{k-1, j} + (ZD_{k-1, j} \times TT_i)) \forall k > 1, i, k' > 1, j \tag{57 – 3, 59 – 3}
\end{align}

\begin{align}
&ZZ_{kij} \geq M_i \sum_{k} Z_{ki}^{i} + \sum_{k'} Z_{k'j}^{i} - 1 \\
&ZZ_{kij} \leq \sum_{k'} Z_{k'i}^{i} \forall k, i, j, j' \neq j' \tag{60 – 3:62 – 3}
\end{align}

\begin{align}
&VV_{jjc} \geq V_j + V_jc - 1 \\
&VV_{jjc} \leq V_jc \forall j, j' \neq j' \tag{63 – 3:65 – 3}
\end{align}

\begin{align}
&ZZV_{kijj} \geq ZZ_{kij} + V_jc - 1 \\
&ZZV_{kijj} \leq V_{j} \forall k, i, j, j' \neq j' \tag{66 – 3:68 – 3}
\end{align}

\begin{align}
&ZZVV_{kijj} \geq ZZ_{kij} + V_jc - 1 \\
&ZZVV_{kijj} \leq V_{j} \forall k, i, j, j' \neq j' \tag{69 – 3:71 – 3}
\end{align}

\begin{align}
&ZZVD_{kijj} \geq D_{jj} - M(1 - ZZV_{kijj}c) \\
&ZZVD_{kijj} \leq D_{j} \forall k, i, j, j' \neq j' \tag{72 – 3:74 – 3}
\end{align}
The design of this model involved firstly examining the proposed model with the assumption of certain input parameters. The basic parameters, including available capacity of machines, intra-extracellular materials and components transport cost and processing time are hypothetical in nature. In this regard, the proposed model has been developed in a hypothetical situation, thus the formation of cells and facility layout are involved in cell manufacturing systems.

Model Development under Random Conditions

The design of this model involved firstly examining the proposed model with the assumption of certain input parameters. The basic parameters, including available capacity of machines, intra-extracellular materials and components transport cost and processing time are hypothetical in nature. In this regard, the proposed model has been developed in a hypothetical situation, thus the formation of cells and facility layout are involved in cell manufacturing systems.

Model-Solving Approach

The CMS is known in the literature as an NP-hard problem, which becomes even more complex when considering the real-life situations. In this regard, the meta-heuristic algorithms, particularly genetic algorithms, simulated annealing and taboo search are used by researchers. The important role of
techniques such as branch and bound algorithm, cutting boards algorithm, especially the algorithm analysis (such as the Benders decomposition, etc.) should not be overlooked. The can ensure optimal solution. Moreover, the Lagrange release algorithm, the column generation algorithm which reach the optimal solution in certain circumstances are essential.

In this regard, the design of this model serves to apply a rigorous approach to solving the proposed model which is a linear programming model involving the genetic algorithm in particular, so as to more appropriately evaluate and assesses the efficiency of meta-heuristic approaches for solving larger-scale problems.

Figure 8-1: Darwin's theory model

In short, genetic algorithm can be derived from the natural coping process, which has been summarized in the table below.

Table 8.1: Comparison of genetic algorithm with the process of natural evolution

<table>
<thead>
<tr>
<th>Processes compatible with natural systems</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome</td>
<td>Numerical fields 0 and 1</td>
</tr>
<tr>
<td>Hereditary characteristics of living creatures are encoded on chromosomes.</td>
<td>The possible solutions are encoded through the numerical sequences.</td>
</tr>
<tr>
<td>Environment</td>
<td>Fitness function</td>
</tr>
<tr>
<td>The principle of natural selection</td>
<td>The problem is included in the form of a mathematical equation.</td>
</tr>
<tr>
<td>Survival criteria of the organism and its reproduction are adaptation to the environment.</td>
<td>Reproduction</td>
</tr>
<tr>
<td>Intersection</td>
<td>Each string is considered as a dependent fitness variable, the fitness value of each string is calculated, and new population is selected proportional to the level of fitness. Therefore, those strings with high fitness are maintained and those with low fitness are eliminated.</td>
</tr>
<tr>
<td>The pairing and splitting of two chromosomes and gene shift between two chromosomes</td>
<td>Intersection</td>
</tr>
<tr>
<td>Mutation</td>
<td>The population strings are paired two by two intersected at one point. Then, the half fragments are replaced between the two strings.</td>
</tr>
<tr>
<td>Replacement of genes with other genes in the DNA chain</td>
<td>Mutation</td>
</tr>
<tr>
<td>The creation of new generations and evolution of creatures.</td>
<td>Replication of the above steps from the proliferation</td>
</tr>
</tbody>
</table>
The Schematic Structure of the Genetic Algorithm

Figure 9.1: The structure of a genetic algorithm.

Numerical Example

Regarding the model solved in LINGO, tables 4-1 to 4-4 illustrate the parameter values of the model, while figures 4-1 to 4-2 display how the machines are laid out. The scheduling and operational sequence of pieces of each machine has been shown in figures 4-1 and 4-2 in detail.

Table 1-10: Parameter values of the model

<table>
<thead>
<tr>
<th>Q</th>
<th>UMC</th>
<th>LMC</th>
<th>KM</th>
<th>KP</th>
<th>C</th>
<th>M</th>
<th>P</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-10: Parameters of each piece

<table>
<thead>
<tr>
<th>TT</th>
<th>CO</th>
<th>CI</th>
<th>dd</th>
<th>Ply</th>
<th>Parameter Piece No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>20</td>
<td>10</td>
<td>100</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>5</td>
<td>80</td>
<td>200</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3-10: Parameters of each machine

<table>
<thead>
<tr>
<th>W</th>
<th>L</th>
<th>Parameter Machine No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4-10: Parameters of each cell

<table>
<thead>
<tr>
<th>UY</th>
<th>LY</th>
<th>UX</th>
<th>LX</th>
<th>Parameter Machine No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>12</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

Matrix $T_{kij}$ is then obtained:

$$
T_{kij} = \begin{pmatrix}
10 & 0 & 0 \\
0 & 20 & 0 \\
0 & 0 & 25
\end{pmatrix}
$$

The Computational Results in Lingo

$Z_{11}^{1,1}, z_{12}^{1,2}, z_{21}^{2,2}, z_{22}^{1,3} = 0$  
$V_{3,1}, V_{2,1}, V_{1,3} = 1$  
$CTM_{1,1} = 10$  
$CTM_{2,1} = 20CTM_{2,2} = 58.5CTM_{1,3} = 55$  
$CTP_1 = 58.5CTP_2 = 55$  
$C_{\text{max}} = 58.5$  
$Z = 6047.5$  

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Figure 10-1: Shows the scheduling and sequence of time/machine operations

Figure 10-1 illustrates the hypothetical scenario of time/machine scheduling and sequence operation prior to application of parameter changes. The machines (M1, M2, M3) in 2 pieces (P1, P2) and 3 pieces (C1, C2) and the number of piece operations (K1, K2) have been displayed.

With changes on the value of TT (time shift per unit of distance on piece i), the impact on the scheduling and layout is examined:

\[ Z_{11}^{11}, Z_{12}^{12}, Z_{21}^{22}, Z_{22}^{22} = 0 \]

\[ V_{3,1}, V_{2,2}, V_{1,3} = 1 \]

\[ CTM_{3,1} = 10\quad CTM_{2,1} = 20CTM_{2,2} = 45CTM_{1,3} = 45 \]

\[ CTP_1 = 45CTP_2 = 45 \]

\[ C_{max} = 45 \]

\[ Z = 4615 \]
As can be seen, making even minor changes in the model parameters, not only affects the scheduling but also how the machines are laid out (Figure 10-2 and Figure 10-2)

**The Proposed Genetic Algorithm Solution**

One of the important components of the genetic algorithm is the structure of chromosomes. Since the solution to the problem needs to cover all the variables of the model, the chromosome structure is as follows.

As indicated in Figure 4.3, the structure of the chromosome is multi-string as many as the available machines. The length of each chromosome $k'm + 2$ entail two distinct parts including two genes **Layout** information and **Scheduling** information entailing $k'm$ genes.

In the Layout, the values are displayed as real numbers that represents the coordinates of the center of gravity of the machine. The second section of Scheduling is displayed as a permutation based on machine operation order for the string of chromosomes. The values of each gene in this section entail the information on piece $i$ and $k$th operation, which are represented by a decimal number $ki$. The real part of these values reflects the number of operation for pieces $i$ and the decimal part represents the number of each piece.
The raw scores are at this stage sorted in ascending order and the ranking of each individual is obtained. Then, the value of $\frac{1}{\sqrt{n}}$ is determined for each individual.

Values of $\frac{1}{\sqrt{n}}$ are within the range $[0,1]$, with the proportionality factor specified.

$$\alpha \times \sum_{n=1}^{10} \frac{1}{\sqrt{n}} = 10 \rightarrow \alpha \times 5.021 = 10 \rightarrow \alpha = 1.99$$

The values of $\frac{1}{\sqrt{n}}$ are multiplied by proportionality coefficient so as to obtain the scale values.

This function has the advantage that prevents the spread of raw scores, which is clearly visible in the example above (Xiaodan, 2007).

The Termination Condition
The termination condition in the proposed algorithm is the completion of certain number of replications depending on the length of the chromosomes, which in turn will be influenced by the size of the problem.

The Computational Results of Genetic Algorithm
The algorithm proposed in this study was coded and solved through MATLAB programming language. Table 5-9 presents some of the results of this algorithm along with the solutions obtained through Lingo.
Table 10-8: illustrate the results of calculation and comparison of algorithm through Lingo

<table>
<thead>
<tr>
<th>No.</th>
<th>Number of pieces</th>
<th>Number of machines</th>
<th>Machine operation</th>
<th>Number of cells</th>
<th>Lingo solution</th>
<th>Algorithm solution (GA)</th>
<th>Operation time (Lingo)</th>
<th>Operation time (GA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>6047</td>
<td>6047</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>8660</td>
<td>8660</td>
<td>95</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>10255</td>
<td>10255</td>
<td>6690</td>
<td>221</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>-</td>
<td>13691</td>
<td>360000</td>
<td>1350</td>
</tr>
</tbody>
</table>

From No. 3 onwards, the optimum solution was not obtained by Lingo and after the time period, the program ceased and the current solution was applied.

**RESULTS AND DISCUSSION**

**Results**

The results showed that the cell configuration and machine layout in cells and the manufacturing space and manufacturing scheduling led to higher productivity through reduction of costs incurred by piece shifts, system maintenance (which depends on total operation time) and delay fees in delivery of pieces. One of the advantages of this model is that it completes all the steps together in one go. As was noted above, the step-by-step procedure can drive us away from the general optimal solutions. The advantages of this model include the following:

1. Given the sequence of operations for pieces.
2. Considering the machine and cell size in a continuous spatial model.
3. Considering the time and cost separately for shifting each piece in real-world applications. Because the pieces differ in terms of weight, volume, fragility, etc. Moreover, they incur various cost and time per shift. The cost and time depend on the distance traveled, which were considered in the model.
4. Considering the importance of each piece that must be delivered on time.

Other results of the study may be pointed out as the results of the proposed algorithm, i.e. the above algorithm managed to provide acceptable solutions as compared to Lingo which took longer time. This in turn reflected the relatively better performance of the proposed method.

**Suggestions for Future Research**

The subjects recommended for future research include:

- A model where the demand of each piece is as many as desired and the delivery deadline for each number is specified.
- The processing unit of each operation is authorized on one machine.
- There is an alternative route for each piece.
- The preparation time of machines depends on the sequence of pieces.
- The dynamic cellular system is planned.
- Other meta-heuristic methods are proposed to improve the solutions are develop large-scale models.
- Using multi-objective models for the purposes of the objective function.
- Stage wise insertion of resource allocation to the above three steps.
- Considering the operation time under potential or fuzzy conditions.
- Considering the sensitivity analysis on parameters.

**REFERENCES**


Research Article


