

MODELING TWO-STAGE INVENTORY CONTROL SYSTEM CONSISTING OF VENDOR AND BUYER UNDER THE STATE OF RANDOM DEMAND AND MACHINE DOWNTIME AND OPTIMIZATION OF MACHINE

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ABSTRACT

Control and inventory management has been regarded as one of the most important operational programs at the organization. Inventory management process influences the liquidity in a company such that its storage beyond the inventory can reduce the liquidity and influence most of decisions. Some studies have stated that 10% to 20% of the assets in a company develop the inventory of the products existing in the company. On the other hand, an organization should have sufficient inventory levels in the growth and development visions so as to meet the customers' satisfaction and provide sufficient support for rest of activities in the organization. Hence, inventory management and determination of optimal policies are the challenges faced in the organization, for which a large body of studies has been conducted. In this study, an inventory model for a system consisting of a seller and a purchaser is developed, through which they pursue the optimal policy of inventory control. In this system, customers' demand is probable, that sometimes there is not the possibility for timely production of product due to machine downtime and unavailability, leading to loss of sale. Further, after modeling this system with the aforementioned assumptions, a heuristic algorithm is proposed which specifies the variables of decision.

Keywords: *Single-vendor Inventory Control System, Potential Demand, Machine Failure, Participation in the Chain, Heuristic Algorithm*

INTRODUCTION

To date, numerous studies have been conducted at the area of inventory management of a two-stage supply chain. The specific hypotheses concerning the issue under study have been conducted at these studies, that the required models must be developed under changing conditions in the problems available in the real world, that their validity must be measured in sake of resolving time. One of the secrets in successful business in modern production management lies on reducing the cost for ordering. This has been drawn into attention by most of scholars (Proteus). Proteus proposed the concept of reduction in ordering cost for the first time and developed a framework for investment for the purpose of reduction of ordering cost in Economic Order Quantity (EOQ) Model. The proposed model by him has become an incentive for most of the scholars in examination of this cost (Taleb zade). Proteus developed Ben-Daya's model by considering the demand in random and changing the delivery time in a linear form to the extent of orders. An & Jo developed Supply Chain Single Vendor – Single Buyer Inventory Model by considering the demand in a fuzzy method. Further, Taleb *et al.*, developed Supply Chain Single Vendor – Single Buyer Inventory Model by delivery time and demand of products. In this model, Fuzzy Delivery Time was considered via the order size and a percent of the not meeting demand over a given period as backlog demand in next periods. The purchaser is in charge for the transport costs. To resolve the proposed algorithm, two Particle Swarm Optimization and Artificial and Bee Colony Hybrid Algorithm were used. An assumption that is used in inventory control models and remained far from the reality relies on this fact that the products which are produced have total quality. Nonetheless, there is this possibility that the production process undergoes decline and causes production of defective products with low quality (Proteus). It can modify effect of defective products in basic EOQ model and examine effect of investment in reduction of defective products on the cost of inventory system (Chang). Chang developed EOQ model by considering the cost at manufacturing unit corresponding to the demand and

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also by considering the production process with defective products. He modeled the inventory decision making as a geometrical plan and obtained an optimal solution for it. Rozenblat *et al.*, and his colleague examined effect of defective products on the order size. Salameh & Jaber developed EOQ model under the assumption of defective products and assumed that the defective products are sold after being monitored.

Nowadays, due to complexity in production processes and increasing competition in the world market, uncertainty is inevitable in an inventory control system. This problem is an often-used problem in the real world, so that there are numerous factors that a supplier due to machine downtime or intrusion in some parts of the production processes does not enable to provide the products of the purchasers, under which the purchasers' dissatisfaction and probable losses to the supplier or purchaser will be followed. In an early overview, it can provide security to cope with unwanted changes by considering the inventory in a sufficient amount. Nonetheless, safety inventory must be determined by taking accurate considerations and considering various effective parameters in the model so as to avoid imposing Carrying cost of inventory to a large extent. Hence, representing an integrative model to consider the definite and probable costs of inventory system can come effective to determine an optimal policy. In this study, overview of Single- buyer and single-vendor model is considered. There is a supplier who provides the required goods for a purchasing company. The supplier is provided with a production process that often fails to supply the required goods in the company due to machine downtime. The time for availability of the machine or machine malfunction is considered in form of two scenarios of uniform distribution and exponential distribution. On the other hand, the purchasing company faces a potential demand to sell the products, under which if the supplier fails to provide the required goods in the company in an appropriate time, the company will have lost sale for the goods. The purchasing company can avoid shortage of good and the cost of lost sale by ordering more goods than the period during which there is no machine downtime in exposure to downtime in the supplier's machine. Nonetheless, more demand for goods can increase the maintenance cost at the purchasing company. Hence, at this state, there must be a model to obtain optimal order and production strategy for the buyer and supplier company undergoing the give-and-take which is fulfilled between various costs such as ordering, maintenance, lost sales and backlog demand.

MATERIALS AND METHODS

Research Method

The proposed model is modeled under the framework of existing standards. Then, using the theory of probability, the convex programming and convex optimization, the optimal value is extracted for the parameters in the model. Thus, the function of model regarding the applied instances is examined and finally the research hypotheses are examined, i.e. improvement in function of chain under the proposed model in the state undergoing participation than the purchasing and selling models is considered. Hence, in this study, the research method is based on data collection and mathematical modeling. Further, after acquiring the optimal response and decision variables, the validity of proposed model via simulation software matlab is considered. This software paves the way to define various modules such as warehouses, manufacturing process and material flow between segments and to consider a variety of probable events such as random inputs and various probable distributions for processing. After some hours from the implementation, it can pursue the state at the balanced and sustainable mode of system. The statistical population consists of all the two-stage inventory control systems in a supply chain including manufacturer and purchaser who face machine downtime and demand. The purchasing company uses the policy 'point of order' for inventory control that gives to the amount of Q order in case the company's inventory level reaches to a certain amount (S). Here, we assume the delivery time of goods from the supplier to the purchasing company as a linear function from the order amount (Q). The purchasing company's demand during a year enjoys a normal distribution with mean and standard deviation. Here, we assume that if the purchasing company's demand is not met on time, the fines due to backlog demand will be imposed on the purchasing company. Hence, the purchasing company faces the

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risk under lack of providing the demand due to the demand excessive than the safety inventory at delivery time. On the other hand, the supplier company specifies its optimal production policy regarding the estimation for mean of demand by the purchasing company and the amount of order (Q) and the costs which are imposed on it. The costs imposed on the supplier company include the costs for preparation and maintenance. Here, we want to develop an integrative model for the supplier and purchasing company so as to minimize sum of costs under the participation conditions. Hence, in this study, we want to obtain economic order quantity and the safety inventory(s) for the purchasing company and determine the optimal number of production cycles corresponding to Q for the supplier company. In this study, we firstly develop model of buyer-seller under their collaboration. After modeling the buyer-seller costs, we develop the integrative model in form of the costs for all the chain per unit time. This model will be a non-linear model with integer number variables, that it cannot expect convex model for the objective function. Hence, a heuristic method is developed which will start from an early response and strive for improvement in previous responses during various repetitions. This method has been used in the literature review for inventory control in a large extent. To validate the proposed model, several scenarios are considered that model the policies of supplier and purchasing company under lack of collaboration, so that the results obtained through the proposed model are compared with the results of these scenarios.

Simulation of the Proposed Model

In simulation of system, the values of uncertain parameters are calculated by using Monte Carlo simulation that the total cost function is obtained at a certain period of time. Indeed, in the proposed model, the cost function was generalized based on time unit. In this simulation, discrete events are considered per day. Hence, the parameters in the sample problem must be expressed per day. Then, followed by implementation of the proposed model for several time in which any repetition in model comes to realize during one year, the value for cost function is obtained per any repetition and the average cost in various repetitions is calculated. If this average value be equal to the value of cost function which was calculated via the mathematical and algebraic expectation relationships, it can say that the developed cost relationships are authentic in the proposed model. To simulate the considered system, the parameters in the inventory control system are modified as shown in table 1.

Table 1: Parameters of inventory control system for discrete system simulation

Unit of measurement	Value	Parameter
Unit/day	80=19200/240	P
Unit/day	20=4800/240	D
\$/cycle	800	A_v
\$/cycle	25	A
\$/unit/day	0.025= 6/240	h_v
\$/unit/day	0.03=7/240	h_b
\$/unit/day	10	c_b
Unit/day	240✓ 6.45=100/	σ
Day	5	L
\$/batch	10	F

The purchaser's order in day t which is shown via $D(t)$ enjoys a normal distribution with mean (D) and standard deviation (σ). The purchaser's inventory level at the end of day t is shown via $I_2(t)$ and the seller's inventory level at the end of day t is shown via $I_1(t)$. The rate of production in the supplier company per day is shown via $p(t)$. Further, two other variables affect the purchaser's and seller's inventory level, that is, one of these variables is shown via $Q_1(t)$ which indicates an amount of the seller's inventory which is sent to the purchaser in day t; another variable is $Q_2(t)$ which indicates an amount of the seller's inventory for the purchaser which is sent to the purchaser in day t. it should be noted that the variables $Q_1(t)$ and $Q_2(t)$ have the same nature, yet they are shown via two separate variables as they

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occur at different periods of time. At the end of day t , in case the purchaser's inventory level reaches to a predetermined amount of R , the purchaser orders the amount of Q . the amount of Q is obtained through the aforementioned models. This order is sent to the supplier company, that is, the seller sends the order to the amount of Q to the purchaser in case the seller's inventory level at the end of day $(t+1)$ is equal to Q . in case the purchaser's inventory is under Q , the order to the amount of $\min(Q_1, I_1(t) + p(t+1))$ is sent to the purchaser at the end of day $(t+1)$. This order at time $(t+L+1)$ will be added to the purchasing company's inventory level. In case the inventory level equals to zero at the supplier company under no machine downtime, the machine will engage in production during $\lceil n.Q / P \rceil$. After ending the machine production, the machine might undergo downtime or needs repair, whereby there will be no access to the machine. As stated, unavailability to machine is probable. Here, the unavailability time regarding probable distribution for machine downtime is produced in random, under which the machine fails to produce. Hence, if the supplier company's demand time reaches to zero during this time and a new demand reaches to the company, this demand will be considered as the lost sale. With the aforementioned explanations, the relationships below will be represented for the purchaser's and seller's inventory level. By introducing variables Q_1 and Q_2 , it can represent the purchaser's and seller's inventory level per day as follow:

$$I_1(t) = \max(0, I_1(t-1) + p(t) - Q_1(t));$$

$$I_2(t) = \max(0, I_2(t-1) - D(t) + Q_2(t));$$

Simulation under the State of Uniform Distribution for the Machine Failure

By considering optimal values Q and n in simulation model, results of simulation model under 10 repetitions will be as follow in table 2:

Table 2: Results of simulation model under the state of uniform distribution for the machine failure in 10 repetitions

	Repeti tion 1	Repeti tion 2	Repeti tion 3	Repeti tion 4	Repeti tion 5	Repeti tion 6	Repeti tion 7	Repeti tion 8	Repeti tion 9	Repeti tion 10
Average total cost	14364	15336	15700	17153	17655	16252	17907	15732	14627	16027

Average value is obtained equal to 16265.44. It can observe that the total cost which was obtained from the proposed model has a difference equal to $681 = 16946 - 16265.44$ or 4%. Hence, it can deduce that the calculations under uniform time distribution in machine failure are authentic.

Simulation under the State of Exponential Distribution for the Machine Failure

By considering optimal values Q and n in simulation model, results of simulation model under 10 repetitions will be as follow in table 3:

Table 3: Results of simulation model under the state of exponential distribution for the machine failure in 10 repetitions

	Repeti tion 1	Repeti tion 2	Repeti tion 3	Repeti tion 4	Repeti tion 5	Repeti tion 6	Repeti tion 7	Repeti tion 8	Repeti tion 9	Repeti tion 10
Average total cost	14352	15123	15460	25461	19732	17281	18931	23044	13677	19854

Average value is obtained equal to 18729. It can observe that the total cost which was obtained from the proposed model has a difference equal to $1772 = 20201 - 18729$ or 8%. Hence, it can deduce that the calculations under exponential time distribution in machine failure are authentic.

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Analysis of Susceptibility of the Proposed Model

In this section, by changing the value of parameters in the proposed model, it is observed what change will come to realize in the optimal value of objective function and the variables. The parameters which have been selected for this purpose include the cost for preparation of production, maintenance cost, ordering cost, transportation cost, cost of lost sale, average demand, the standard deviation of demand, and the parameters related to probability distribution of machine downtime. Six values are considered for each of the parameters. Concerning the value for the parameters, here +50% implies that new value for parameter is considered equal to 1.5 times to the parameter's value.

Analysis of Susceptibility of the Proposed Model under the time Undergoing Machine Downtime at the Uniform State

Effect of changing parameters on the value of objective function and the variables in decision in the proposed model under uniform distribution in machine downtime will be considered. Results of susceptibility of the proposed model at this state have been characterized in tables 4-6.

Table 4: Analysis of the total cost of susceptibility of proposed model under the time undergoing machine downtime at the uniform state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
18151.08	17559.06	16946.34	16255.37	15185.47	K
19620.63	18332.36	16946.34	15436.22	13761.16	h_v
18252.21	17611.23	16946.34	16254.62	15429.84	h_b
16985.28	16965.82	16946.34	16926.84	16907.31	A
19645.48	18345.66	16946.34	15420.41	13725.65	c_b
233.4.07	20197.18	16946.34	13543.59	9963.334	D
16875.18	16910.81	16946.34	16981.78	17017.13	σ
	16946.34	16946.34	16946.34	16946.34	b
16946.34					
16977.5	16961.93	16946.34	16930.74	16915.12	F
19654.45	18350.15	16946.34	15415.93	13716.68	L
16830.11	16906.04	16946.34	17013.41	17087.54	α

Table 5: Analysis of the optimum size of the order of susceptibility of the proposed model under the time undergoing machine downtime at the uniform state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
824.2336	797.3234	769.4727	927.8529	866.716	K
664.4978	711.2396	769.4727	844.8374	947.8084	h_v
714.7176	740.5813	769.4727	802.0327	1101.491	h_b
771.2429	770.3593	769.4727	768.5861	767.6985	A
892.1606	833.0783	769.4727	700.1122	623.0778	c_b
1135.909	949.6111	769.4727	594.534	423.2082	D
765.8303	767.6537	769.4727	771.2875	773.098	σ
769.4727	769.4727	769.4727	769.4727	769.4727	b
770.8892	770.1813	769.4727	768.7635	768.0537	F
892.1606	833.0783	769.4727	700.1122	623.0778	L
763.5248	767.4097	769.4727	772.9077	776.7063	α

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Table 6: Analysis of the optimum size of n of susceptibility of the proposed model under the time undergoing machine downtime at the uniform state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
2	2	2	1	1	K
2	2	2	2	2	h_v
2	2	2	2	1	h_b
2	2	2	2	2	A
2	2	2	2	2	c_b
2	2	2	2	2	D
2	2	2	2	2	σ
2	2	2	2	2	b
2	2	2	2	2	F
2	2	2	2	2	L
2	2	2	2	2	α

Analysis of Susceptibility of the Proposed Model under the time Undergoing Machine Downtime at the Exponential State

Effect of changing parameters on the value of objective function and the variables in decision in the proposed model under exponential distribution in machine downtime will be considered. Results of susceptibility of the proposed model at this state have been characterized in tables 7-9.

Table 7: Analysis of the total cost of susceptibility of the proposed model under the time undergoing machine downtime at the exponential state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
20728.92	20464.3	20199.41	19923.06	19618.97	K
23722.16	22040.58	20199.41	18107.56	15801.14	h_v
21347.87	20780.49	20199.41	19529.99	18815.27	h_b
20215.98	20207.69	20199.41	20191.12	20182.83	A
24259.06	22291.41	22291.41	17909.01	15239.19	c_b
38019.87	28319.39	22291.41	15806.12	13926.09	D
20139.65	20169.54	22291.41	20229.25	20259.08	σ
18663.49	19335.41	20318.7	21677.45	24089.34	λ
20225.93	20212.67	20199.41	20186.14	20172.88	F
22267.77	21255.93	20199.41	18936.02	17509.48	L
20101.93	20165.54	20199.41	20255.95	20318.7	α

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Table 8: Analysis of the optimum size of the order of susceptibility of the proposed model under the time undergoing machine downtime at the exponential state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
635.3165	619.1664	603.6778	672.0155	657.2583	K
609.5745	646.1083	603.6778	647.3103	661.5797	h_v
580.8595	592.0498	603.6778	703.3927	720.2535	h_b
604.6272	604.1522	603.6778	603.204	602.7308	A
750.1753	634.1733	603.6778	559.321	488.8727	c_b
850.8958	643.0285	603.6778	908.0479	1187.313	D
559.8345	601.7513	603.6778	605.614	607.56	σ
674.4506	669.9687	611.4817	640.1126	629.8813	λ
605.198	604.4371	603.6778	602.92	602.1637	F
811.9832	745.4687	603.6778	538.9828	419.7519	L
597.431	601.494	603.6778	607.3547	611.4817	α

Table 9: Analysis of the optimum size of n of susceptibility of the proposed model under the time undergoing machine downtime at the exponential state

+50%	+25%	0%	-25%	-50%	The percent for changing the parameter
	5	5	5 4	4	K
	4	4	5 5	6	h_v
	5	5	5	4	h_b
	5	5	5	5	A
4		5	5	5	c_b
3	4		5 3	2	D
	5	5	5 5	5	σ
	4	4	5 5	6	λ
	5	5	5 5	5	F
	4	4	5 5	6	L
5	5	5	5	5	α

Discussion

Concerning overview of the literature review at this area, it was observed that some hypotheses have been added in the model used in the related works that the developed model is close to the conditions governing the real world; however, considering new hypotheses might complicate the proposed models. Further, it was observed that one of the existing gaps at this area might lie on considering the period of machine downtime in producing the product at the supplier's company. Further, the probability nature of demand in the aforementioned system causes difficulty at modeling of this system in a way that it reflects the conditions governing the real world. Hence, a new model was developed in which these hypotheses are mentioned. To develop these models regarding the probability of time of a production period, renewal theory was used. Average costs at a period are obtained in using this theory, in which the obtained costs are divided into the average period of time so as to acquire the average cost per time unit. Using Markov and probability models renewal theory, it can indicate that such an approach obtains a bias estimation for the average costs at time unit. After developing model via the considered hypotheses, two heuristic algorithms were proposed for each of uniform and exponential distributions at machine downtime. Due to non-convex resolving space, it was mentioned that these algorithms fail to extract the optimal response for the proposed model and indicated that the results from the proposed algorithm are authentic.

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Conclusion

For this purpose, firstly the models associated to lack of collaboration were developed for each of the scenarios of uniform distribution and exponential distribution at the machine downtime. Using tables 4, 5 and 6, it can extract the results below:

- per an increase at the cost for preparation of production, average total cost increases.
- per an increase at maintenance cost for both purchaser and supplier, average total cost increases.
- per an increase at ordering cost, average total cost increases.
- per an increase at lost sale, average total cost increases.
- per an increase at demand rate, average total cost increases.
- per an increase at standard deviation of demand, average total cost decreases
- per an increase and decrease at top bound in machine downtime in uniform distribution to the extent of $\pm 50\%$, it can observe that no change comes to realize at average total cost and decision variables. The reason for this is explained in this way that the optimal response for the proposed model is acquired:

$$TC = \frac{K + h_v \cdot \frac{nQ^2}{D} \left(\frac{n}{2} - \frac{nD}{2P} + \frac{1}{2} \right) + n \cdot h_b \left(\frac{Q^2}{2D} + \frac{S \cdot Q}{D} \right) + A + F \cdot n + c_b \cdot n \cdot b(S, L)}{\frac{n \cdot Q}{D}}, T_d \geq b$$

In this equation, when downtime comes to realize in a machine at a cycle and goes beyond the top bound for machine downtime, thus parameter b will not exist in the average cost equation, whereby there will be no effect in average costs and value of variables.

- per an increase at transport cost, average total cost increases.
- per an increase at delivery time, average total cost increases.
- per an increase at the level of providing service or parameter α , average total cost decreases.

Further, delivery time of goods, the cost for preparation, the cost for maintenance of purchaser and seller are mentioned in next priorities. Clearly, it can observe that changing rest of parameters has no effect on average total cost.

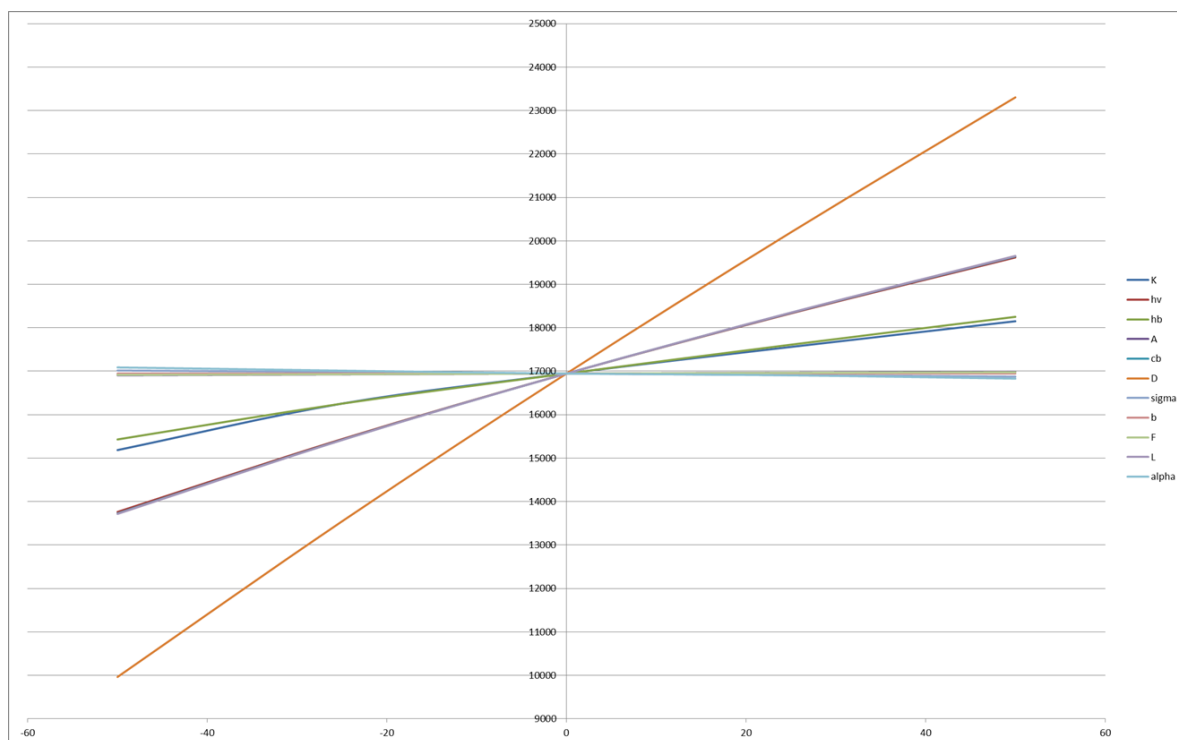


Figure 1: The diagram representing effect of various parameters on total cost under collaboration and machine downtime at the uniform state

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Using tables 7, 8 and 9, it can refer to the results as follows:

- per an increase at the cost for preparation of production, average total cost increases.
- per an increase at maintenance cost for both purchaser and supplier, average total cost increases.
- per an increase at ordering cost, average total cost increases.
- per an increase at lost sale, average total cost increases.
- per an increase at demand rate, average total cost increases.
- per an increase at standard deviation of demand, average total cost decreases
- per an increase at average machine downtime period in exponential distribution or an increase in parameter λ , it can observe that average total cost decreases.
- per an increase at transport cost, average total cost increases.
- per an increase at delivery time, average total cost increases.
- per an increase at the level of providing service or parameter α , average total cost decreases.

According to figure 2, it can observe that the average demand has the highest effect on average costs.

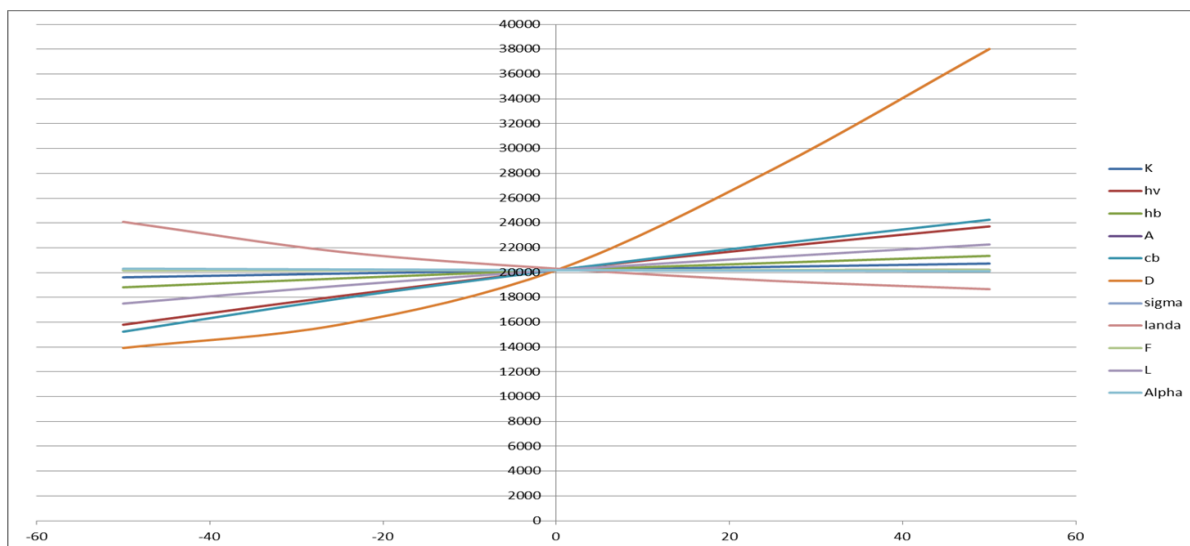


Figure 2: The diagram representing effect of various parameters on total cost under machine downtime period at an exponential state

Suggestions at Upcoming Studies

However single-vendor single-buyer integrated inventory systems have been examined in numerous related works, the factors below are suggested for upcoming studies regarding the results of this research:

- Development of more accurate algorithms for resolving the proposed model
- As mentioned, the heuristic method is the proposed solution that its optimality is not assured. Hence, meta-heuristic algorithms can be developed for the model, resulting in high-quality responses.
- Associating the safety inventory with the rest of parameters including economic order quantity and the parameters associated to time of machine downtime
- developing the proposed model for several products
- developing the proposed model for supply chain with higher levels
- considering the production rate and acquiring the rate of optimal production
- developing the model with more limitations such as capacity of warehouse, considering the defective items

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