*Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231– 6345 (Online)*
*An Open Access, Online International Journal Available at www.cibtech.org/sp.ed/jls/2015/03/jls.htm*
*2015 Vol. 5 (S3), pp. 1276-1283/Sasan et al.*
***Research Article***

# DETERMINATION OF UPPER BOUND LIMIT ANALYSIS OF THE COEFFICIENT OF LATERAL PASSIVE EARTH PRESSURE IN THE CONDITION OF LINEAR MC CRITERIA

**Ghasemloy Takantapeh Sasan, *Akhlaghi Tohid and Bahadori Hadi**
*Department of Civil Engineering, Urmia Branch, Islamic Azad University, Urmia, Iran*
**Author for Correspondence*

## ABSTRACT
The determination of the lateral earth pressure of a fill on a retaining wall when frictional forces act on the back fill of the wall is one of the classical stability problems in soil mechanics. These problems can quite conveniently be solved by limit analysis methods. This method includes the theory of upper and lower bound. Using these two theories, one can obtain a range of loading in which the actual collapse load is located. This range is obtained by minimizing upper bound and maximizing lower bound. In this paper, to evaluate the coefficients of lateral passive earth pressure, upper bound limit analysis theory is used in which initially, by optimization of formulation, coefficient of lateral passive earth pressure due to weight, cohesion and surcharge are determined. For optimization, optimization functions available in MATLAB are utilized and finally, effect of parameters such as backfill slope, wall inclination on coefficient of lateral passive earth pressure is investigated. Results demonstrate that aforesaid parameters have a significant contribution to the coefficient of lateral passive earth pressure.

***Keywords:*** *Limit Analysis, Upper Bound, Lateral Passive Earth Pressure, Retaining Wall, and Linear MC Criteria*

## INTRODUCTION
Passive resistance evaluation is necessary for design of many geotechnical structures such as retaining walls, sheet piles, bridge abutments, anchor blocks, and group pile caps. Therefore, the problem of determining coefficient of lateral earth pressure is an important issue in geotechnical engineering. Previously, many researchers attempted to develop and elaborate techniques for evaluating the earth pressures (Yang, 2007a). The main methods already developed can be classified into four groups: (1) the limit equilibrium method (Donald & Chen, 1997), (2) the slip-line method (Roth & Oxley, 1972), (3) the limit analysis method (Yang & Yin, 2005; Yang & Zou, 2006a), and (4) the numerical methods (Kim *et al.*, 1997). Among them, the limit equilibrium method is one of the traditional analytical approaches, includes Rankin, Coulomb and Log-Spiral methods.
The Rankin method considers a smooth wall and the resultant passive is inclined at angle equal to the angle of surface inclination behind the wall. In Coulomb's approach, the soil-wall friction angle is supposed to take a value between zero and the internal friction angle of the backfill material. In this method, a simple equilibrium is used for calculating resultant passive force. Both methods are developed for granular material and are based on the assumption of plane failure surfaces. However, it is generally discovered that the assumption of a plane failure surface is not reasonable for rough walls. There are especially so far passive cases in which, Coulomb's method may yield increasingly non-conservative (i.e., unsafe) predictions as the value of soil-wall friction angle increases. To overcome such shortcoming, the Log-Spiral method was developed (Terzaghi, 1943; Terzaghi *et al.*, 1996). Caquot and Kerisel (1948) summarized tables and charts for passive pressure coefficient based on this method for cohesion-less soil and simple geometries. Recently, Duncan and Mokwa (2001) also developed an Excel spreadsheet computer program based on the Log-Spiral method which can accommodate both cohesive and frictional soils, although it is limited to level ground, a vertical wall, a uniform surcharge, and homogeneous soil.
Although conventional displacement finite element (FE) analysis can be used for prediction of passive resistance of soils (e.g., Potts and Fourier, 1986; Day and Potts, 1998) such estimates are not rigorous bounds on the true value. As an alternative method which can provide numerical results having high

quality compared to the method of limit equilibrium is the method of limit analysis. This is a strong mathematical method which provides an exact estimation of the upper and lower bound with regard to an actual collapse load. Contrary to the limit equilibrium and slip-line, this method takes the stress – strain relationship of the soil into account with an ideal method. This idealization is called normal (flow rule) (Chen, 1975).

Till now, a linear Mohr-Coulomb (MC) yield criterion is widely used in the limit analysis of stability problems. The reason is probably that the linear MC yield criterion can be expressed as a circle in the stress space for a plane strain condition.

This characteristic allows us to approximate the circle by a fracture surface, which is a linear function of the stresses of plane strain problems. Thus, based on the upper and lower bound theorems, the stability problems are formulated as linear programming ones.

In this paper, the method of upper bound analysis is used for obtaining coefficients of lateral passive earth pressure and for this purpose; Log-Sandwich failure mechanism is used. In this type, failure mechanisms is determined by considering the linear yield criterion of Mohr – Coulomb, rate of external work and internal energy dissipation into account and then taking both as equal, formulation of lateral passive earth pressure in three cases of weight, cohesion and surcharge are determined. Since in the theory of upper bound analysis, the goal is to minimize the objective function, for optimization of the function, *fmincon* optimization function available in MATLAB library is utilized.

***Description of Limit Analysis Method***

The method of limit analysis addressed in the form of limit theorems, is the generalization of the principle of maximum plastic work (Hill, 1948). Drucker *et al.*, (1952) introduced the limit theorems using generalization of them. Assumptions used in the limit analysis methods are as follows: 1) behavior of material are completely plastic, 2) limit case is expressed by a function in the form of $F(\sigma_{ij}) = 0$ which is called yield function. Representation of the function in main stresses space is known as yield surface, 3) plastic behavior of material is a function of plastic flow rule. In other words, $\dot{\varepsilon}_{ij}^{p} = \lambda \frac{\partial F(\sigma_{ij})}{\partial \sigma_{ij}}$, where, $\dot{\varepsilon}_{ij}^{p}$ is the tensor of plastic relative displacement velocity, $\sigma_{ij}$ is the tensor of stress and λ is a non-negative scalar function.

Application of upper and lower bound in the method of limit analysis is a powerful for determination of the limit of collapse load in stability problems of soil mechanics. In lower bound theorem, problem is solved using definition of assumed stress fields. Stress field used in this case must satisfy: 1) the equilibrium equations, 2) stress boundary conditions, 3) nowhere violate the yield criterion. According to this theorem, under each stress field satisfying above conditions, free plastic flow will not occur. In other words, external load of the stress field will not exceed actual collapse load.

In the theorem of upper bound, by considering an assumed velocity field and taking internal and external forces as equal, collapse load will be computed.

According to this theorem, if the assumed velocity field satisfies the velocity boundary conditions and compatibility conditions, calculated load will be higher than actual one. Be selecting appropriate stress and velocity fields and making solutions of both methods close to each other, one can find a range in which the actual collapse load is located.

In problems in which solution of both methods are the same, actual solution is found. In this way, in this method, location of each solution compared to actual solution is known and above and below approximation is clear. Strength and robustness of the limit analysis method is in this note. In present paper, the method of upper bound analysis is used and in what follows, utilized mechanism and it algorithm are explained.

*Utilized Formulation and Mechanism*

General model considered for the Log-Sandwich mechanism is illustrated in Figure 1. This mechanism shows a logarithmic spiral shearing zone, OBC, sandwiched between two rigid blocks, OAB and OCD. In this mechanism, the velocities $V_1$ and $V_3$ for the rigid triangles OAB and OCD are assumed perpendicular to the radial lines OB and OC, two angular parameters $\rho$ and $\psi$ describe the mechanism completely.

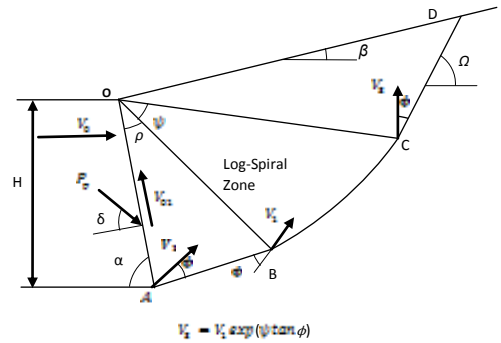


**Figure 1: Log-Sandwich failure mechanism**

Compatibility velocity diagram for this case is represented in Figure 2.

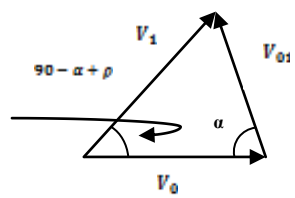


**Figure 2: Compatibility velocity diagram**

For finding the formulation of the coefficients of lateral passive earth pressure due to weight, cohesion and surcharge, after taking external work and internal energy dissipation as equal, we have:
The rate of external work due to self-weight for OAB, OBC and OCD zones is:

$$-\frac{\frac{1}{2}\gamma H^2 V_1 \sin\rho\cos(\rho-\varphi)\cos(\alpha-\rho)}{\sin^2\alpha\cos\alpha} \quad (1)$$

$$\frac{-\frac{1}{2}\gamma H^2 V_1\cos^2(\rho-\varphi)}{\sin^2\alpha\cos^2\varphi(1+9\tan^2\varphi)}\{\cos(\alpha-\rho)[-3\tan\varphi+(3\tan\varphi\cos\psi+\sin\psi)\exp(3\psi\tan\varphi)]+$$
$$\sin(\alpha-\rho)[1+(3\tan\varphi\sin\psi-\cos\psi)\exp(3\psi\tan\varphi)]\} \quad (2)$$

$$\frac{-\frac{1}{2}\gamma H^2 V_1\cos^2(\rho-\varphi)\sin(\alpha-\rho-\psi+\beta)\cos(\alpha-\rho-\psi)\exp(3\psi\tan\varphi)}{\sin^2\alpha\cos\varphi\cos(\alpha-\rho-\psi+\beta+\varphi)} \quad (3)$$

The external work done by the components of the resultant wall load *P* moving in the horizontal direction with velocity $V_0$ is equal:

$$P_{pn}V_0[\sin\alpha+\tan\delta\cos\alpha] \quad (4)$$

Since a cohesionless soil is being considered, the only dissipation occurs at the soil-wall interface. The dissipation by sliding friction is equal:

$$P_{pn}\tan\delta\, V_{01} \quad (5)$$

And finally

$$K_{p\gamma}=\frac{\sec\delta}{\sin\alpha+\tan\delta\cos\alpha-\frac{\tan\delta\cos(\alpha-\rho)}{\cos\rho}}\{\frac{\tan\rho\cos(\rho-\varphi)\cos(\alpha-\rho)}{\sin\alpha\cos\varphi}+\frac{\cos^2(\rho-\varphi)}{\cos\rho\sin\alpha\cos^2\varphi(1+9\tan^2\varphi)}[\cos(\alpha-$$
$$\rho)[-3\tan\varphi+(3\tan\varphi\cos\psi+\sin\psi)\exp(3\psi\tan\varphi)]+\sin(\alpha-\rho)[1+(3\tan\varphi\sin\psi-$$
$$\cos\psi)\exp(3\psi\tan\varphi)]]+\frac{\cos^2(\rho-\varphi)\sin(\alpha-\rho-\psi+\beta)\cos(\alpha-\rho-\psi)\exp(3\psi\tan\varphi)}{\cos\varphi\sin\alpha\cos(\alpha-\rho-\psi+\varphi+\beta)\cos\rho}\} \quad (6)$$

Similarly, coefficients of lateral passive earth pressure due to cohesion and surcharge can be expressed as follows. Of course, it should be kept in mind that for the sake of brevity, rate of external work and internal energy dissipation for above cases are not stated and only the final formulation is presented.

$$K_{pc} = \frac{\sec\delta}{\sin\alpha+\tan\delta\cos\alpha-\frac{\tan\delta\cos(\alpha-\rho)}{\cos\rho}}\{\tan\rho+\frac{\cos(\rho-\varphi)\sin(\alpha-\rho-\psi+\beta)\exp(2\psi\tan\varphi)}{\cos\rho\cos(\alpha-\rho-\psi+\varphi+\beta)}+ + \frac{\cos(\rho-\varphi)[\exp(2\psi\tan\varphi)-1]}{\sin\varphi\cos\rho}\} \quad (7)$$

$$K_{pq} = \frac{\sec\delta}{\sin\alpha+\tan\delta\cos\alpha-\frac{\tan\delta\cos(\alpha-\rho)}{\cos\rho}}\{\frac{\cos(\rho-\varphi)\cos(\rho+\psi-\alpha)\exp(2\psi\tan\varphi)}{\cos\rho\cos(\alpha-\rho-\psi+\varphi+\beta)}\} \quad (8)$$

Distribution of $K_{p\gamma}$, $K_{pc}$ and $K_{pq}$ has two variables of $\psi$ and $\rho$ and these functions must be minimized with respect to available constraints. As stated earlier, for optimization of available functions, optimization available in *fmincon* is used.

*Comparison and Analysis*

In this section, upper bound of the lateral passive earth pressure exerted to retaining wall will be investigated from various aspects. This paper covers contributing factors such as backfill slope and wall inclination. Furthermore, numerical results of this research are compared with other works.

## RESULTS AND DISCUSSION

Results obtained for $k_p$ in case of $\phi = 40^\circ$ is compared with that of other works and shown in table 1. For the case of smooth wall, $\frac{\delta}{\phi} = 0$, value of $k_p$ obtained by any method is 4.6. However, if the wall friction increases, coefficient of lateral passive earth pressure increases as well. The important implication is that both the Log-Spiral and limit equilibrium methods by Caquot and Kerisel (1948) and Duncan *et al.*, (2001) predict higher values of $k_p$ except for the fully rough case ($\frac{\delta}{\phi} = 0$). In addition, it can be said that results of this work are slightly different from finite element limit analysis of Shiau *et al.*, (2008) which is negligible. In fact, it means that using failure mechanism of Log-Sandwich which solely utilizes limit analysis and doesn't use finite element method, we can obtain more exact results.

Using upper bound analysis method, wider range of analysis can be found for different values of $\phi$. Numerical results of the analysis are represented in table 2. In general, numerical results of upper bound analysis method provide an appropriate estimation of coefficients of lateral passive earth pressure.

**Table 1: Comparison of results ($\alpha = 90^\circ, \beta = 0^\circ, \phi = 40^\circ$)**

| $\delta/\phi$ | $k_p$ | | | | | |
|---|---|---|---|---|---|---|
| | **Coulomb Theory** | **Caquot and Kerisel (1948)** | **Log-Spiral Method (Duncan et al., 2001)** | **Sokolovski (1960)** | **Finite element Lmit Analysis Method (Shiau *et al.*, 2008)** | **Upper Bound (This Paper)** |
| 0 | 4/60 | 4/59 | 4/60 | 4/60 | 4/61 | 4/59 |
| 1/3 | 8/15 | 8/13 | 8/17 | - | 7/79 | 7/71 |
| ½ | 11/77 | 10/36 | 10/50 | 9/69 | 10/03 | 10/06 |
| 2/3 | 18/72 | 13/10 | 13/08 | - | 12/87 | 13/07 |
| 1 | 92/72 | 17/50 | 17/50 | 18/20 | 20/10 | 20/9 |

**Table 2: The coefficient of lateral passive earth pressure based on this paper**

| $\delta/\phi$ | $k_p$ | | | | | |
|---|---|---|---|---|---|---|
| | $\phi = 20^\circ$ | $\phi = 25^\circ$ | $\phi = 30^\circ$ | $\phi = 35^\circ$ | $\phi = 40^\circ$ | $\phi = 45^\circ$ |
| 0 | 2/03 | 2/46 | 3.00 | 3/69 | 4/59 | 5/82 |
| 1/3 | 2/39 | 3/08 | 4/05 | 5/49 | 7/71 | 11.39 |
| ½ | 2/58 | 3/43 | 4/70 | 6/71 | 10/06 | 16/18 |
| 2/3 | 2/77 | 3/81 | 5/44 | 8/16 | 13/07 | 22/84 |
| 1 | 3/16 | 4/62 | 7/10 | 11/67 | 20/91 | 41/94 |

Results obtained from optimization of the formulation corresponding to $k_{p\gamma}$ can be summarized for better illustration of contributing factors as follows:

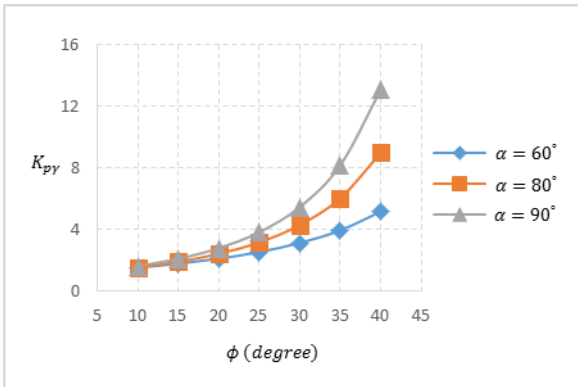


**Figure 3: Changes in $k_{p\gamma}$ in terms of $\phi$ for different $\alpha$ in Log-Sandwich failure mechanism ($\beta = 0^\circ$ , $\delta = \frac{2}{3}\phi$)**

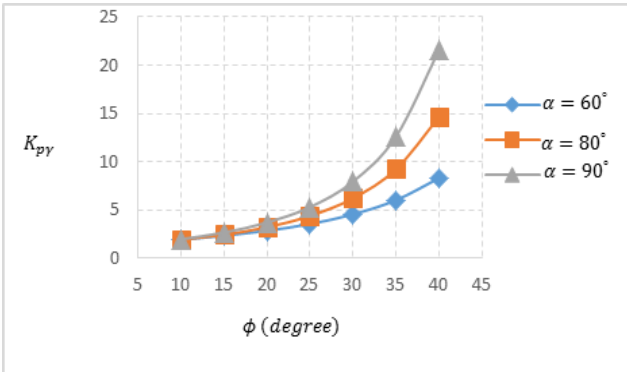


**Figure 4: Changes in $k_{p\gamma}$ in terms of $\phi$ for different $\alpha$ in Log-Sandwich failure mechanism ($\beta = 10^\circ$ , $\delta = \frac{2}{3}\phi$)**

Method of limit analysis using Log-Sandwich mechanism can provide coefficients of lateral passive earth pressure as a result of cohesion and surcharge and effect of various factors on this coefficient can be shown as follows:

Effect of wall inclination: by considering $\delta = \frac{2}{3}\phi$ and $\beta = 0^\circ$, we can illustrate changes in $K_{pc}$ and $K_{pq}$ for various $\phi$ and α as shown in Figs. 5 and 6. As can be seen, by increasing α from $60^\circ$ to 90°, values of $K_{pc}$ and $K_{pq}$ increases and for $K_{pq}$ it can be claimed that for $\phi$ = 10°, value of the coefficient for $60^\circ$, $80^\circ$

and 90° are 1.51, 1.51 and 1.57, respectively which are nearly the same, by considering higher values for $\phi$, value of these coefficients changes.

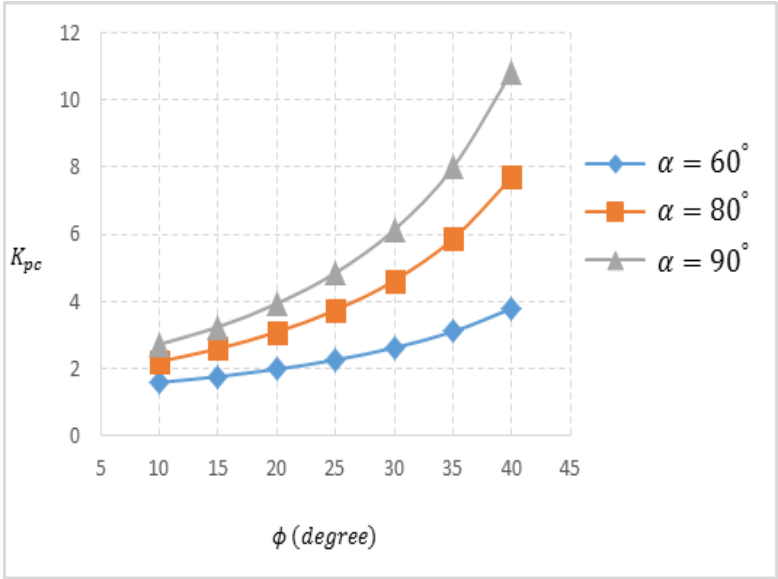


**Figure 5: Changes in $k_{pc}$ in terms of $\phi$ for different $\alpha$ in Log-Sandwich failure mechanism ($\beta = 0^\circ, \delta = \frac{2}{3}\phi$)**

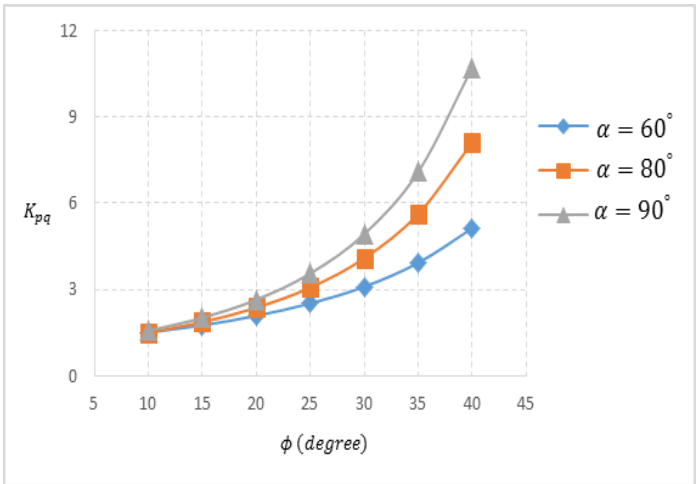


**Figure 6: Changes in $k_{pq}$ in terms of $\phi$ for different $\alpha$ in Log-Sandwich Failure mechanism ($\beta = 0^\circ, \delta = \frac{2}{3}\phi$)**

Effect of backfill slope: numerical results obtained from above analysis, are limited to level ground. In this section of research, we can observe the effect of backfill slope on coefficient of lateral passive earth pressure due to cohesion and surcharge. Figs. 7 and 8 show the changes in $K_{pc}$ and $K_{pq}$ in terms of $\phi$ and various $\beta$s. In these figures, $\delta = \frac{2}{3}\phi$ and $\alpha = 90^\circ$ are considered. By taking $\beta$ as much as $0^\circ$, $10^\circ$ and 15°, it can be said that as $\beta$ increases, for constant $\phi$, $K_{pc}$ and $K_{pq}$ increase as well. The reason is clear. For higher $\beta$, weight of soil behind the wall increases and this leads to an increase in coefficients of lateral passive earth pressure.

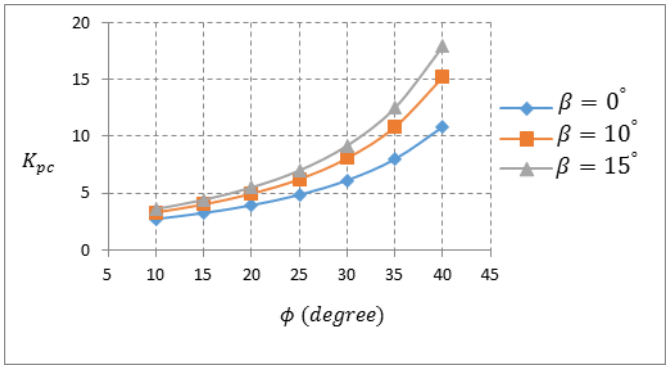


**Figure 7: Changes in $k_{pc}$ in terms of $\phi$ for different $\beta$ in Log-Sandwich Failure mechanism ($\alpha = 90^{\circ}$, $\delta = \frac{2}{3}\phi$)**

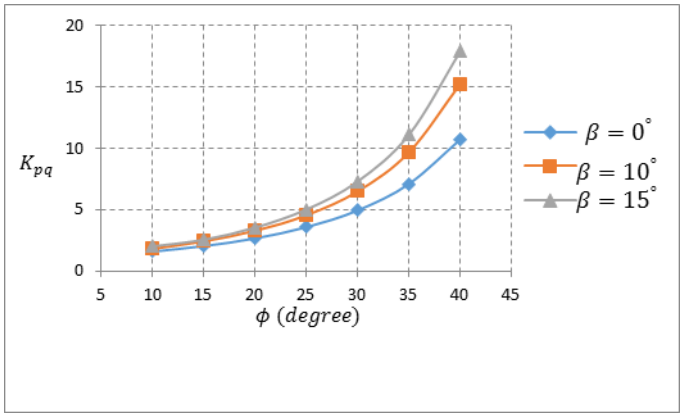


**Figure 8: Changes in $k_{pq}$ in terms of $\phi$ for different $\beta$ in Log-Sandwich failure mechanism ($\alpha = 90^{\circ}$, $\delta = \frac{2}{3}\phi$)**

***Conclusion***

Solution obtained using limit analysis method is provided for complementing literature. By taking the Log-Sandwich failure mechanism into account and through optimization function of *fmincon* and factors contributing to coefficients such as wall inclination and backfill slope, it can be claimed that:

1) By applying the method of upper bound method, values obtained for coefficient of lateral passive earth pressure is lower compared to other methods.

2) Current method can yield coefficients of lateral passive earth pressure as a result of cohesion, surcharge and weight soil.

3) Results obtained from optimization of formulation corresponding to coefficients of lateral passive earth pressure due to cohesion, surcharge and weight soil represent the increase of aforesaid coefficients with respect to internal friction angle for wall inclination and backfill slope.

In conclusion, it can be said that presented method can be applied as a reliable method for design purposes.

**REFERENCES**

**Caquot A and Kerisel J (1948).** *Tables for the Calculation of Passive Pressure, Active Pressure and Bearing Capacity of Foundations*, Gauthier-Villars, Paris.

**Chen WF (1975).** *Limit Analysis and Soil Plasticity* (Elsevier) Amsterdam.

**Day RA and Potts DM (1998).** The effects of interface properties on retaining wall behavior. *International Journal for Numerical and Analytical Methods in Geomechanics* **22** 1021-1033.

**Donald IB and Chen Z (1997).** Slope stability analysis by the upper bound approach: fundamentals and methods. *Canadian Geotechnical Journal* **34**(6) 853-862.

**Drucker DC, Prager W and Greenberg HT (1952).** Extended limit design theorems for continuous media. *Quarterly of Applied Mathematics* **9**(4) 381-389.

**Duncan JM and Mokwa R (2001).** Passive earth pressures: theories and tests. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE* **127**(3) 248-257.

**Hill R (1948).** A theory of the yielding and plastic flow of anisotropic metals. *Proceedings of the Royal Society of London A* **193** 281-297.

**Kim KW, Lee WY and Sin H (1997).** A Cnite-element analysis for machining with the tool edge considered. *Journal of Materials Processing Technology* **86** 45-55.

**Potts DM and Fourie AB (1986).** A numerical study of the effects of wall deformation on earth pressures. *International Journal for Numerical and Analytical Methods in Geomechanics* **10** 383-405.

**Roth RN and Oxley PLB (1972).** Slip-Line Celd analysis for orthogonal machining based upon experimental Eow Celd. *Journal of Mechanical Engineering Science* **14** 85-97.

**Sokolovski V (1960).** *Statics of Soil Media* (Butterworths Publications) London.

**Terzaghi K (1943).** *Theoretical Soil Mechanics* (John Wiley and Sons, Inc.) New York.

**Terzaghi K, Peck RB and Mezri G (1996).** *Soil Mechanics in Engineering Practice*, 3rd edition (John Wiley and Sons, Inc.) New York.

**Yang XL and Yin JH (2005).** Upper bound solution for ultimate bearing capacity with a modified Hoek-Brown failure criterion. *International Journal of Rock Mechanics and Mining Sciences* **42**(4) 550-560.

**Yang XL and Yin JH (2006a).** Linear Mohr-Coulomb strength parameters from the nonlinear Hoek-Brown rock masses. *International Journal of Non-Linear Mechanics* **41**(8) 1000-1005.

**Yang XL (2007a).** Upper bound analysis of active earth pressure with different fracture surface and nonlinear yield criterion. *Journal of Theoretical and Applied Fracture Mechanics* **47**(1) 46-56.