Indian Journal of Fundamental and Applied Life Sciences ISSN: 2231– 6345 (Online)
An Open Access, Online International Journal Available at www.cibtech.org/sp.ed/jls/2015/01/jls.htm
2015 Vol.5 (S1), pp. 4122-4128/Janghorbani et al.

Research Article

NONLINEAR PHENOMENA IN POWER ELECTRONIC: CHAOTIC BEHAVIOR OF BUCK CONVERTERS

Mohammadreza Janghorbani, *Shahram Javadi and Masoud Khosravi
Islamic Azad University, Central Tehran Branch, Tehran, Iran
**Author for Correspondence*

ABSTRACT
Power electronic is rich in nonlinear dynamic. Its operation is characterized by cyclic switching of circuit topologies, which gives rise to different nonlinear behaviors. Among power electronic converters, DC/DC buck converters have been studied with constant-frequency pulse-width modulation feedback control in continuous conduction mode. Time-domain and phase-space plots for several periodic and chaotic orbits are presented. Bifurcation diagram is studied together with periodic orbits and chaotic behavior of the circuit. Two simulation methods including exact solution and EMTP simulation are used and the importance of accurate modeling is justified. Also a method for computation of Lyapunov exponents in discontinuous systems is reviewed and implemented.

Keywords: *Bifurcation Diagram, Buck Converter, Chaos, Lyapunov Exponents, Symbolic Analysis, Transient Simulation*

INTRODUCTION
Power electronics is a discipline spawned by real-life applications in commercial, industrial, aerospace and residential environments. Much of the development of the field of power electronic revolves around some immediate needs for solving specific power conversion problems. DC/DC converters that are used to regulate and step down (buck converters), step up (boost converters), or both (buck-boost or Ćuk converters) are among the most-widely used power electronic circuits. It can be said that one of the most important characteristics of power electronic circuits is their highly nonlinear behavior. This nonlinearity is due to both nonlinear elements used in these devices (e.g., diodes, BJTs, transformers, and control circuitry employed such as comparators and pulse-width modulators) and the switching operation, which changes the topology of the circuit (Di Bernardo and Vasca, 2000), (Yu *et al.*, 2012).
The behavior of an electrical circuit can be characterized in its steady-state or in the transient state. In its steady state, an electrical circuit can exhibit one of the following four behaviors (Hamill *et al.*, 1992): (1) point stability, (2) cycle stability, (3) instability (but saturated), and (4) chaotic stability. In point stability, the circuit currents and voltages settle down to a constant value. The circuit is called stable in this case, and representation of the system in phase-space is a single point.
Most circuits are designed to operate in this mode. In cycle stability, the circuit states repeat themselves as periodic functions of time with a single period of T, period T and its multiples, or some disproportionate period. An oscillator circuit is perhaps the most used example of this type of behavior. (Yan-Li *et al.*, 2006)
In chaotic stability, the dynamical system is divergent but its trajectory is bounded. This fourth class is called chaotic behavior and such trajectory is called strange attractor that arises in many power electronic converters, for example buck converter, boost converter, and the ripple regulator circuit (Ruzbehani *et al.*, 2006), (Wang *et al.*, 2013).
Existence of chaos in power electronic circuits has received great attention during last two decades. The most studied power electronic circuits are DC/DC converters due to their simpler structure. Chaotic behavior of buck converters has been studied in (Fossas and Olivar, 1996), (Kinsner, 2003).
This paper presents a study of the chaotic behavior of the buck converter and its mathematical modeling. Two methods for simulation of the buck converter, including the exact solution, and simulation in PSCAD/EMTDC program are presented and results are compared. Lyapunov exponents are defined and calculated. Some final remarks in last section conclude the report.

Research Article

Chaotic Dynamics

Chaotic operation is the fourth class of stability of a dynamical system. A continuous system governed by a set of at least three nonlinear, first-order, differential equations with no external input (autonomous), or of lower order but with an external input such as time (non-autonomous), can exhibit chaotic behavior.

The signals resulting from a chaotic system, although aperiodic, are bounded. The behavior of a system is referred to as chaotic if the trajectory of its states possesses three properties. First, it should show high sensitivity to the initial conditions. Even the smallest changes can lead to very large differences in the trajectory, although the chaotic system is governed by a set of completely deterministic equations and even in the absence of noise.

The second property of chaos is the underlying process of folding. While trajectories do not intersect, so they are limited to a certain area-the strange attractor.

The third characteristic of chaos is mixing, which means trajectories, regardless of the initial conditions, will eventually reach everywhere in the phase-space. A more formal definition is that for any two open intervals of non-zero length, a value from one interval maps to another point in the other interval after a sufficient number of iterations.

Two types of diagrams are frequently used in the study of chaotic systems: the phase-space diagram and the bifurcation diagram. The phase-space diagram, which is an n-dimensional diagram while n is the number of states of the autonomous system, which shows the state trajectory of the system.

There are several methods to characterize chaos. The largest Lyapunov exponent and the information dimension are among them. The largest Lyapunov exponent and the information dimension for the studied buck converter are 0.64 and 2.21, respectively.

The Second-Order Buck Converter

The buck converter is one of the simplest but most useful power converters: A step-down power electronic converter that converts an unregulated DC voltage to a lower DC voltage regulated by means of closed-loop feedback operation.

The circuit diagram of the buck converter is illustrated in figure 1.

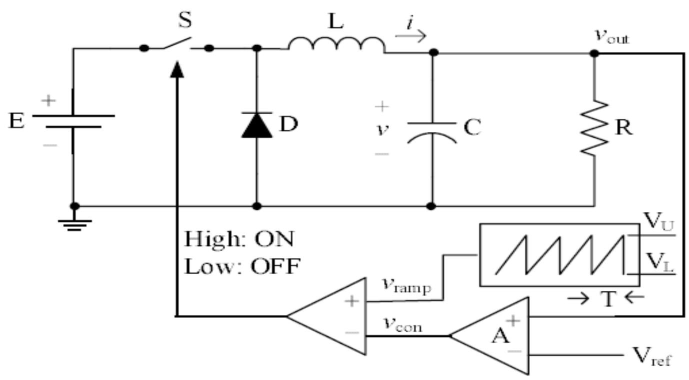

Figure 1: Schematic of the second-order buck converter with simplified control circuitry

Behavior of the Circuit

There are two switches in a second-order buck converter. One switch is uncontrolled (diode D) and the other one (S) is controlled by the feedback controller. At each time, only one of these two switches is in the ON state. A capacitor C is connected in parallel with the load to help maintaining a relatively constant load voltage. The series inductor L is used as an energy-storing device.

During the ON state of S, energy from the source E is stored in L. When S is open, the inductor delivers the stored energy to the load R.

The feedback loop tries to keep the load voltage, v_{out}, constant. The load voltage is measured and passed to the subtractor block to form the error signal, v_{con}, which is

$$v_{con} = A(v_{out} - V_{ref}) \qquad (1)$$

Where A is the amplification factor. This signal is then compared with a saw-tooth ramp signal with a minimum of V_L, maximum of V_U and period of T, defines as below:

$$v_{ramp} = V_L + (V_U - V_L) \bmod(t/T, 1) \tag{2}$$

If the magnitude of the saw-tooth signal is greater than that of the error signal v_{con}, S is turned ON, otherwise S remains OFF. This means that the switch state changes whenever $v_{con} = v_{ramp}$ is satisfied.

Circuit Model

At each instant, the system state is determined by the two state variables v (capacitor voltage) and i (inductor current) as well as the state of switch S.

The buck converter can be considered as two circuits multiplexed in time. The differential equations for v and i are:

$$\frac{dv}{dt} = -\frac{1}{RC}v(t) + \frac{1}{C}i(t) \tag{3}$$
$$\frac{di}{dt} = -\frac{1}{L}v(t) + \frac{\zeta(t)}{L}E$$

Where ζ is the control signal and is equal to 1 when the switch is ON and is 0 when the switch is OFF.
The circuit is simulated using the parameters shown in Table 1. Input voltage E is used as the bifurcation Parameter and is varied between 15 and 40 V.

Table 1: Bcuk converter parameters used for simulation

Circuit Parameters	**R(Ω)**	**L(mH)**	**C(μF)**		
	22	**20**	**47**		
Controller Parameters	V_U (V)	V_L (V)	T (μs)	V_{ref} (V)	A
	8.2	3.8	400	11.3	8.4

The differential equations (3) of the circuit are solved in the next section by two methods: the exact closed-form solution, and PSCAD simulation.

The Buck Converter Simulation

Behavior of the closed-loop buck converter is analyzed using three methods. First, the closed-form piecewise solution of the system equations is presented. The extreme sensitivity of the circuit is the main incentive for looking for the exact solution of the circuit, so that the round-off error does not propagate from one step to another and the most accurate results can be obtained. The system equations are also solved by the commercial simulation EMTP-type program PSCAD/EMTDC. In all cases, circuit elements are assumed to be ideal.

Closed-Form Method

In this method, (3) is solved for v(t) and i(t) with a constant ζ and the closed-from solution is obtained. The switching happens whenever the following boundary condition is satisfied.

$$v_{con}(t_c) = v_{ramp}(t_c) \tag{4}$$

Where t_c is the switching time. Then, the equation is solved using Newton-Raphson method with a maximum allowable error of 10^{-10} to find the exact switching time.

For E equal to 24, 28, 32, and 33 V, phase-space plots show period-1, -2, -4, and the chaotic behavior of the system as shown in figure 2. Figure 3 illustrates time-domain waveforms for chaotic operation for E = 33 V. It can be clearly seen that it is possible for vcon to skip some cycles (no switching in a cycle) as well as to intersect the ramp voltage more than once in a cycle (multiple switching in a cycle).

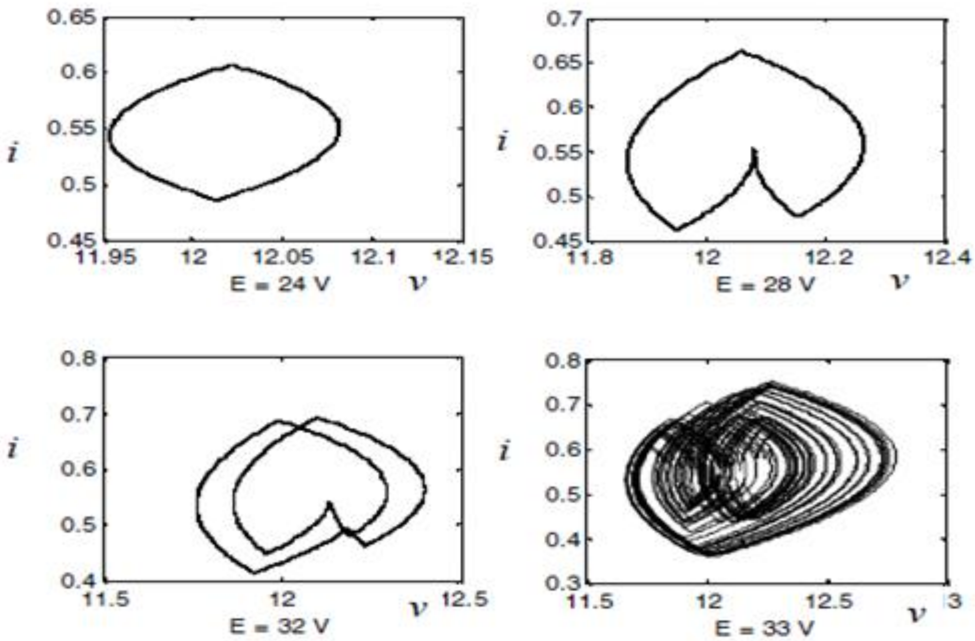

Figure 2: Phase-space diagram of the buck converter showing period-1 (E = 24 V), period-2 (E = 28 V), period-4 (E = 32 V), and chaotic (E = 33 V) waveforms obtained from the exact solution

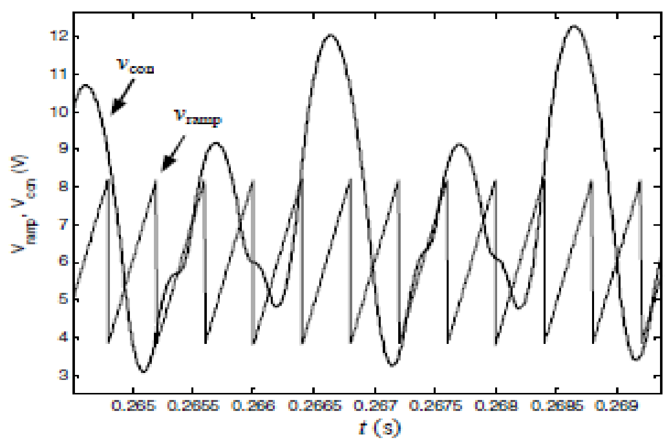

Figure 3: Chaotic operation of the buck converter obtained by exact solution with E = 33 V (shown are v_{con} and v_{ramp})

Bifurcation diagram is plotted for input voltage E swept from 15 to 40 V (figure 4) and is obtained by recording the voltage at the end of each period.

It clearly shows the succession of period doublings.

The separation between period-doubling points decreases with the number of periods. The abrupt transition from the period-doubling to chaotic region is related to the sharp, singular points in the phase-space diagram of the converter.

PSCAD/EMTDC Simulation

The model is also implemented in PSCAD/EMTDC electromagnetic transient simulation program.

Figure 5 shows the converter model. Taking advantage of the interpolation block in PSCAD, t_c is found with an accuracy of 0.01% of time step. Phase-space diagrams for four input voltages values (24, 28, 32, and 33V) are shown in figure 6, which are quite similar to those of the exact solution in figure 2. This is because of the proper selection of time-step as well as approximating the witching instant by interpolation. Figure 7 shows time-domain voltage waveform an input voltage of E = 33V.

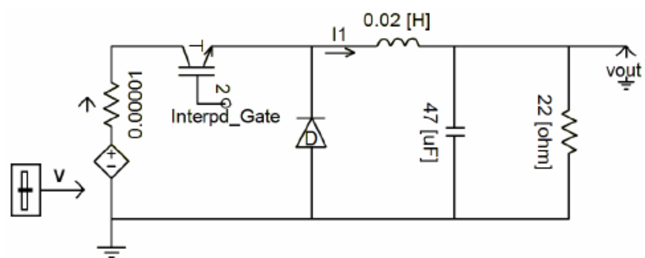

Figure 4: Bifurcation diagram obtained by sampling the output voltage at the end of each cycle

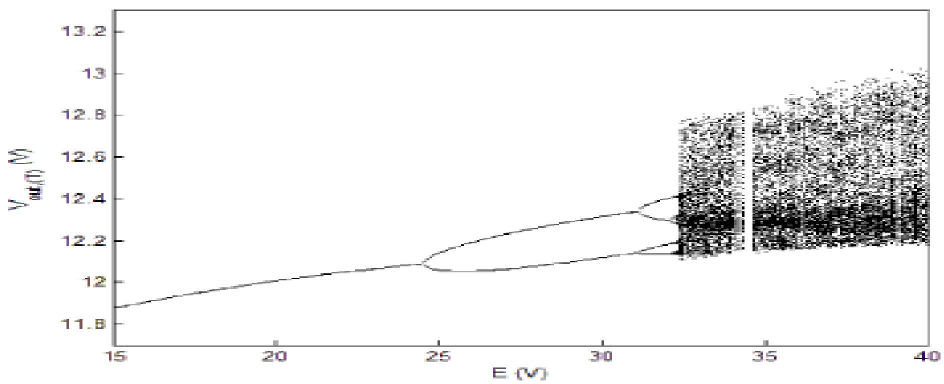

Figure 5: PSCAD model of the buck converter

Lyapunov Exponent (L.e.)

Conceptually the L.e. is a quantitative test of the sensitive dependence on initial conditions of the system. The number of Lyapunov exponents for a system is equal to the dimension of its phase space. Normally the largest exponent is used, because it determines the horizon of predictability of the system. In this sense, the inverse of the largest Lyapunov exponent is called Lyapunov time, which

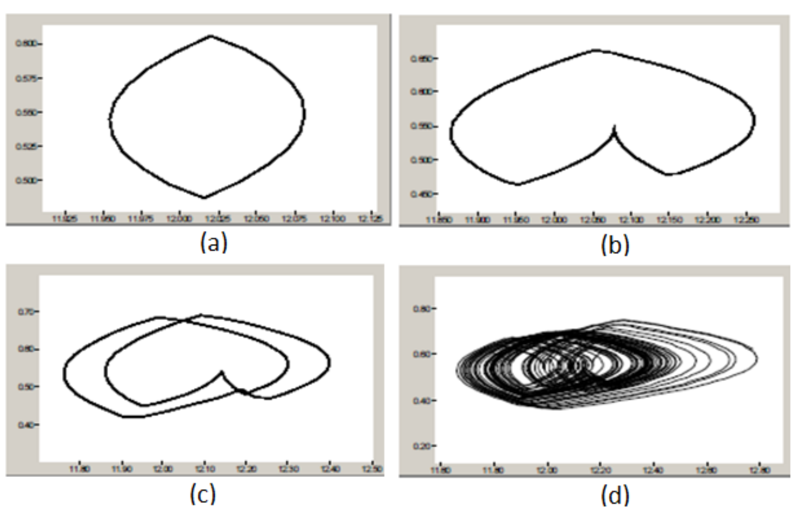

Figure 6: Phase-space diagrams (output voltage on x-axis, inductor current on yaxis) of the PSCAD run for (a) E = 24 V showing periodic operation, (b) E = 28 V showing period-2, (c) E = 32 V showing period-4, and (d) E = 33 V showing chaotic operation

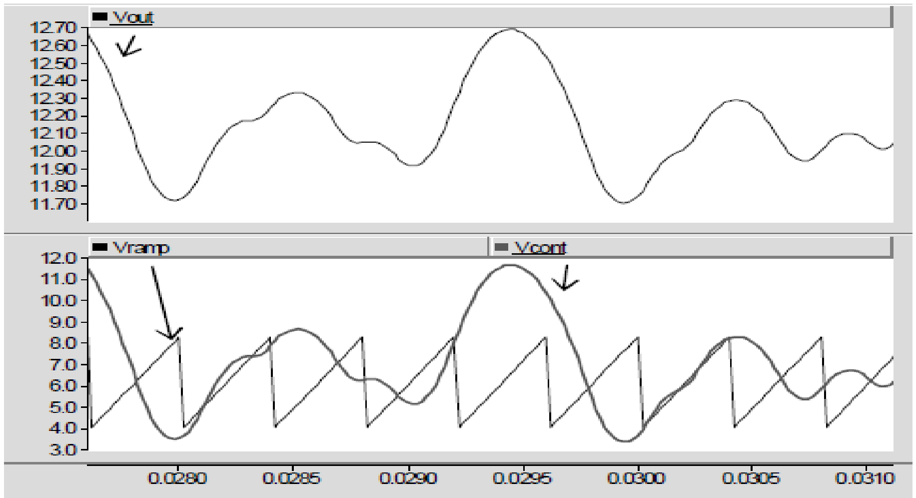

Figure 7: Plot of output voltage, ramp generator output, and the control voltage vs. time for the chaotic operation with E = 33 V

Defines the characteristic folding time of the system. A negative Lyapunov exponent is characteristic of dissipative (non-conservative) systems, which exhibit point stability. The more negative the exponent, the faster the stability. An exponent of $-\infty$ shows the extremely fast convergence, and hence stability. A Lyapunov exponent of zero is characteristic of a cycle-stable system. The orbits maintain their separation in this case. A positive Lyapunov exponent, on the other hand, implies that nearby points, no matter how close, will finally diverge to an arbitrary separation. This happens in the case of instable as well as chaotic system. Distinction between these two is made by using the set of Lyapunov exponents. The largest Lyapunov exponent is defined as

$$\lambda_{max} = \lim_{\delta x(0)\to 0} \lim_{t\to\infty} \left(\frac{1}{t} \ln \frac{\|\delta x(t)\|}{\|\delta x(0)\|} \right) \tag{5}$$

where $\delta x(t)$ is the perturbation of the system.

To overcome the problems in applying the above equation to power electronic circuits, an approximate method has been suggested by Müller (Müller, 1995). This method is used for the buck converter and λ_{max} is calculated from

$$\lambda_{max} = \left(\frac{1}{(t - t_0)/T} \ln \frac{\|\delta x(t)\|}{\|\delta x(t_0)\|} \right) \tag{6}$$

While for E = 24 V, the maximum Lyapunov exponent is $\lambda_{max} = 3\times10^{-4}$ (practically zero) that indicates a stable system, for chaotic region, E = 33 V, $\lambda_{max} = 0.68$, which is a positive number.

CONCLUSION

In this paper, the buck converter and its operation in the chaotic regime is studied using time-domain, phase-space, and bifurcation diagrams, as well as Lyapunov exponents. The chaotic nature of circuit operation intensifies the need for precise determination of the switching instances. Therefore, two methods (analytical solution and simulation in the PSCAD/EMTDC program) are used to study the circuit and find the most suitable combination of simplicity of implementation and accuracy of results. Comparing the results, it is found that simulation in PSCAD/EMTDC, being a simulation program primarily developed for study of rapidly changing phenomena, requires less effort, is generally faster, and offers more flexibility in tailoring the model to include complex converter and control circuitry models. This could establish a new and comprehensive platform to study and detect chaos in power electronic circuits.

Research Article

ACKNOWLEDGMENT
The authors thank Islamic Azad University, Central Tehran Branch for all helps and supports.

REFERENCES
Banerjee S and Verghese GC (2001). *Nonlinear Phenomena in Power Electronics* (IEEE Press Piscataway).
Di Bernardo M and Vasca F (2000). Discrete-Time Maps for the Analysis of Bifurcations and Chaos in Dc/Dc Converters. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* **47** 130-143.
F**eng Y, Guan K and Chen T (2007).** Bifurcation And Chaotic Behavior Of Buck Converter. *Modern Electronics Technique* **20** 055.
F**ossas E and Olivar G (1996).** Study of Chaos in the Buck Converter. *IEEE Transactions on Circuits and Systems I: Fundamental Theory And Applications* **43** 13-25.
Hamill DC, Deane JH and Jefferies DJ (1992). Modeling Of Chaotic Dc-Dc Converters By Iterated Nonlinear Mappings. *IEEE Transactions on Power Electronics* **7** 25-36.
Kinsner W (2003). Fractal and Chaos Engineering Course Notes. Winnipeg, Mb: Dept. Electrical & Computer Eng., *University of Manitoba.*
Müller PC (1995). Calculation of Lyapunov Exponents For Dynamic Systems With Discontinuities. *Chaos, Solitons & Fractals* **5** 1671-1681.
Ruzbehani M, Zhou L and Wang M (2006). Bifurcation Diagram Features Of A Dc–Dc Converter Under Current-Mode Control. *Chaos, Solitons & Fractals* **28** 205-212.
Wang J, Bao B, Xu J, Zhou G and Hu W (2013). Dynamical Effects of Equivalent Series Resistance of Output Capacitor in Constant On-Time Controlled Buck Converter. *IEEE Transactions on Industrial Electronics* **60** 1759-1768.
Yan-Li Z, Xiao-Shu L and Guan-Rong C (2006). Pole Placement Method of Controlling Chaos in Dc–Dc Buck Converters. *Chinese Physics* **15** 1719.
Yu D, Iu HH, Chen H, Rodriguez E, Alarcón E and El Aroudi A (2012). Instabilities in Digitally Controlled Voltage-Mode Synchronous Buck Converter. *International Journal of Bifurcation and Chaos* **22**.