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Research Article

ESTIMATING SATURATED HYDRAULIC CONDUCTIVITY USING FUZZY SET THEORY

***Soulmaz Hamedi**
Department of Soil Science, Science and Research Branch, Islamic Azad University, Tehran, Iran
**Author for Correspondence*

ABSTRACT
Saturated hydraulic conductivity, K_s is a crucial input parameter in modeling flow process in soil. Due to inherent temporal and spatial variability of this parameter, large numbers of samples are required to characterize areas of land. Hydraulic characteristics can be obtained from direct laboratory and field measurements. However, such measurements are time consuming and costly. In this paper, an alternative approach, based on fuzzy set theory, has been proposed to estimate K_s of various soils selected from UNSODA database by using particle size distribution (PSD) as input variables. To achieve this goal, the grain-size distribution has been first transformed from crisp distribution to possibilistic form that is related to fuzzy logic. At the next stage, a fuzzy rule-based system is defined for mapping a set of input data into soil saturated hydraulic conductivity as output. Furthermore, the fuzzy approach has been compared with pedotransfer functions (PTFs) and Rosetta Package. To quantify the prediction accuracy of the fuzzy approach ME, RMSE, EF and CRM were calculated. The results indicated that the fuzzy approach in comparison with derived PTFs and Rosetta could more accurately predict saturated hydraulic conductivity. Therefore, this approach could be applied to improve efficiency of existing models in simulation of soil water flow.

Keywords: *Saturated Hydraulic Conductivity; Particle size Distribution; Fuzzy Set Theory; Fuzzy Rule-based Models*

INTRODUCTION
Soil hydraulic properties are important inputs for most hydrological and flow transport models. But these properties are expensive and time consuming to measure. Due to spatial distribution of soil hydraulic properties, few measurements taken on field for input parameters cannot characterize field conditions. Therefore, several indirect methods (i.e. statistical and empirical models) have been proposed to estimate soil hydraulic properties from easily measured or available soil data. Since the beginning of this century, numerous models have been presented to account for this purpose.

In statistical methods (i.e. PTFs) that have been applied to estimate soil hydraulic properties, quantitative relationships among certain soil properties are usually achieved by using multiple linear regressions. The term pedotransfer function (PTF) was innovated by Bouma (1989). Recently, PTFs have become a popular topic in soil science research. Several types of function have been developed to predict either physical or chemical properties of the soils with different properties (Ghorbani *et al.,* 2010; Khodaverdiloo *et al*., 2011). Various mathematical methods such as multiple linear regressions (Wosten *et al*., 1995) have been used to provide empirical relationships between basic soil properties and parameters to be predicted. Regression models are easy to apply but the accuracy of them is dependent on the number of original data base and the scale of region. Furthermore, these techniques do not consider spatial correlation of soil observations and rely on the assumption of linearity (Verma *et al*., 2009). However, this assumption is in contrast with the complex and vague relationships among soil properties

To take into account the spatial autocorrelation of observed values in field samples, geostatistical methods have also been investigated by some researchers (McBratney and Odeh, 1997). However, geostatistical methods are limited for complex and large area due to the assumption of stationary or large numbers of field observations. These methods are best for modeling water flow in small areas which have enough field observations. Therefore, application of geostatistical methods over large and diverse landscapes introduced significant challenges for researchers.

In recent years, fuzzy set theory seems to be useful approach to overcome some limitations of mentioned above methods. Fuzzy sets describing imprecision and vagueness were coined by Zadeh (1965). The theory of fuzzy sets is mathematically intuitive method of quantifying imprecision and uncertainty by grouping individuals into classes that do not have sharply defined boundaries. Fuzzy sets are useful for describing ambiguity and vagueness in conceptual or mathematical models of empirical phenomena. Fuzzy sets have been applied in different fields such as decision making and control (Dubois and Prade, 1980). In recent years, some studies have been done to incorporate fuzzy theory in another science.

In (1988), Incorporation of fuzzy in geostatistical field was first introduced by Bardossy *et al.*, Fuzzy Kriging is useful tools to overcome the problem of insufficient numbers of measurement by using fuzzy variogram parameters and fuzzy regression techniques (Bardossy *et al.*, 1990a; Bardossy *et al.*, 1990b; Bardossy *et al*., 1990c). In the procedure mentioned above, the input data are represented as fuzzy numbers.

Many applications of fuzzy set theory involve development of fuzzy knowledge for hydrology. Dou *et al*., (1995) used imprecise parameters in groundwater flow simulations (Verma *et al*., 2009). They solve 2D steady state groundwater flow equations by incorporating fuzzy theory in 2D finite difference approximation (Dou *et al*., 1995). In another study, Dixon (2005) carried out groundwater vulnerability by incorporating GIS and fuzzy rules. Bardossy *et al.*, (1995) used fuzzy rules for modeling water flow in the unsaturated zone. Zhu *et al.*, (2010) used soil fuzzy membership values to estimate detailed spatial variation of soil properties. The results of their research showed that the model based on regression with fuzzy membership values works well over area where soil environmental relationships are more complicated. Schulz and Huwe (1999) have chosen the Darcy-Bukingham transport equation to incorporate fuzzy soil hydraulic properties in modeling. The sensitivity and uncertainty analysis of one-dimension steady state water transport are evaluated by using fuzzy set in layered soil profile. Verma *et al.*, (2009) presented a method based on fuzzy set theory to quantify the uncertainty in estimation and to express imprecision of input data. Also, fuzzy set theory is employed by Hu *et al.*, (2003) for identification of soil types and hydraulic properties of contaminated soils. They also used fuzzy reasoning to convert uncertain system inputs to fuzzy linguistic information. Several studies have been introduced in using fuzzy theory for pollution and different remediation policies under imprecise conditions. Also, Ross *et al.*, (2007) used approximate reasoning for estimating soil saturated hydraulic conductivity. Woldt *et al.*, (1996) used fuzzy models for management of diffuse pollution in groundwater.

The objectives of this study were to (1) investigate if fuzzy approach based on fuzzy set theory can be applied to estimate saturated hydraulic conductivity, and (2) to test the accuracy of fuzzy approach in comparison with derived PTFs and Rosetta package.

MATERIALS AND METHODS

Experimental PSD data of the UNSODA hydraulic property database (Nemes *et al.*, 2001) was used in this study. Data sets, including data of K_s, bulk density and particle density of 60 soils with textures ranging from sand to clay, were selected. The 60 undisturbed soil samples with more than four points on the PSD were chosen from the UNSODA. In this study, an alternative approach, based on fuzzy set theory, was proposed to estimate saturated hydraulic conductivity K_s of various soils selected from UNSODA database by using PSD as input variables. The grain-size distribution is theoretically a continuous curve representing the amount various partial size present in soil (Figure 2). They are cumulative distribution functions (CDF), which are different from fuzzy sets. The vertical axis representing fraction of the soil sample (by weight) is less or equal in size to each grain size value along the horizontal axis.

The grain-size distribution must be transformed from an original distribution to fuzzy set form, without changing or losing any information. To achieve this goal, the CDF must first be converted to probability density function (PDF). At first, we have to represent the grain-size distribution mathematically. A two-parameter, log-normal distribution is used to fit grain-size distribution data. If the PSD of a soil is assumed to be lognormal, its cumulative PSD is described as:

$$F(\ln d\ ) = \left[\frac{1}{\sigma_d\sqrt{2\pi}}\right] \exp\left[\frac{-(\ln d\text{-}\mu)^2}{2\sigma_d{}^2}\right] \quad (1)$$

where d is the particle diameter, $F(\ln d)$ the cumulative lognormal distribution function of $\ln d$, μ and σ_d are mean and standard deviation of $\ln d$.
By differentiating Eq. (1), another form can be produced to represent the distribution of particle sizes. The differentiation produces probability density function (PDF) of particle-size distribution. The differentiation form is given in Eq. 2.

$$\mathrm{f}\ (\ln d\ ) = \left[\frac{1}{\sigma_d \ln d\ \sqrt{2\pi}}\right] \exp\left[\frac{-(\ln \ln d\text{-}\mu)^2}{2\sigma_d{}^2}\right] \quad (2)$$

A transformation is used to convert data in probabilistic form to possibilistic form (related to fuzzy logic). Fuzzy sets and possibility distributions are generally defined on the same scale. As such, they are defined on the same range, since their scales are consistent (Ross *et al*., 2007).
The most common transformation is based on the ratio scale is given in Eq. (3) (Salicone, 2007).

$$r_i = \frac{p_i}{\max(p_i)} \qquad i=1,\ldots,n \quad (3)$$

Where,

$$r_i = r(x_i), \qquad p_i = p(x_i)$$

where r_i and p_i are the possibility and probability of element x_i of universal set (X), respectively.
Each probability value is divided by the maximum probability value according to Eq. (3).The results of the transformation of the soil grain PDFs into soil grain PDs for the four soil samples are shown in Figure (3).
After the grain size distributions is transformed from crisp representation (CDF) to fuzzy form (PD), the soil grain type and K_s were defined in the fuzzy set form. Various shape of fuzzy set can be defined for soil grain type and K_s.
The shape of a membership function of fuzzy sets depends on the application. The most commonly used is the liner type, trapezoidal and triangular. In the follow, a fuzzy rule-based system defined for mapping a set of input data into soil saturated hydraulic conductivity as output variable. The fuzzy rule-based system usually consists from four major parts: fuzzification, fuzzy rule base, fuzzy inference engine, defuzzification. There is several fuzzy rule-based system (Fuller, 1995). In this paper, the Mamdani Fuzzy Inference System (MFIS) was employed to map the soil type fuzzy sets to soil saturated hydraulic conductivity fuzzy sets. In the fuzzy rule base of MFIS, three rules are defined. The membership that activates the rule is calculated by plotting the PD curve of the soil samples and soil type fuzzy sets in single figure (Figures 2 and 3).
The PD curve of soil samples may intersect the soil type fuzzy sets in several points, but the maximum intersection is important for us. The maximum point of intersection is the degree that each rule activates. the rules relate soil grain size to saturated hydraulic conductivity. Figures 3 and 4 show how the PD of two samples sample with different in particle percentages intersects each of soil grain size fuzzy sets at several points.
The output of the inference engine is fuzzy set, but the crisp data is required. There are several commonly operators for defuzzification, including centroid of area (COA) method that is often referred to as the center-of-gravity method (COG), smallest of the maximums, max or mean-max membership principles and the weighted-average method. However, the center of gravity, the most commonly operator is selected to defuzzify according to follow equation (Fuller, 1995):

$$COG = \frac{\sum_i \mu_{\mathrm{K}_s}(\mathrm{K}_{s_i})\mathrm{K}_{s_i}}{\sum_i \mu_{\mathrm{K}_s}(\mathrm{K}_{s_i})} \quad (4)$$

where, μ_{K_s} is the membership function of the hydraulic conductivity fuzzy set, and K_{s_i} is a member of K_s
In the follow, to evaluate the fuzzy approach compared with derived PTFs and Rosetta to derive PTFs, the basic soil properties such as the particle size distribution, bulk density, the geometric mean and the geometric standard deviation of particles have been chosen to analyze statistically. The multiple linear regression analyses were performed to correlate the dependent and independent variables of selected data. Since multiple regression approach assumes normal distributions of the dependent variables, the degree of normality of the data was tested with SPSS Software 16.0. The normality tests showed that some of the variables were not normally distributed. Therefore, the data with non-normal distributions were normalized as much as possible. In order to derive the PTFs, the sand, silt and clay contents, the bulk densities, dg and σ were applied as independent variables as well as K_s was the dependent variable. To avoid any multicolinearity, stepwise procedure was used to derive the regression models. Only functions with significant and uncorrelated independent variables ($p < 0.05$) were accepted. Both K_s and the predictor variables σ_g, sand and clay percentages were log-transformed, while the remaining independent variables tested (bulk density, textural fractions) were not. These regression equations and their statistics are showed in Table (4).
The geometric mean d_g (L) and geometric standard deviation σ_g (-) of the soil particle diameters were determined using methods proposed by Shirazi and Boersma (1984):

$$d_g = \exp a, \qquad a = \sum_{j=1}^{3} f_i \ln \mu_i \tag{5}$$

$$\sigma_g = \exp b, \qquad b^2 = \sum_{j=1}^{3} f_i \ln^2 \mu_i - a^2 \tag{6}$$

where j is the number of soil separates (e.g. clay, silt and sand), $\boldsymbol{\mu_i}$ is the mean diameter (in mm) of particles in soil separates i, $\boldsymbol{f_i}$ is the percentage of total mass having a diameter equal to or less than $\boldsymbol{\mu_i}$.
The Rosetta package (schaap *et al*., 2001) includes five hierarchical PTFs to predict the water retention curve, as well as the saturated and unsaturated hydraulic conductivity. The Rosetta apply neural network and bootstrap approach for parameter predication and uncertainty analysis respectively. The hierarchy in PTFs allows prediction of the saturated hydraulic conductivity using limited (soil texture class only) to more extended (texture, bulk density, and one or two water retention points) input data (Khodaverdiloo *et al*., 2011). In this study, The K_s was also predicted with the hierarchical PTFs of Rosetta using sand, silt and clay percentages, as well as the bulk density, of input data.

Model Evaluation

The accuracy and the reliability of the fuzzy approach in comparison with PTFs and Rosetta package were evaluated by using mean error (ME), root mean square error (RMSE), modeling efficiency (EF) and coefficient of residual mass (CRM). The mathematical expressions of the various statistics are as follows (Willmot, 1981; Zarei *et al*., 2011; Ghorbani *et al.*, 2010; Khodaverdiloo *et al.*, 2011):

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N}(P_i - O_i)^2}{N}} \tag{7}$$

$$CRM = \frac{\sum_{i=1}^{n} O_i - \sum_{i=1}^{n} P_i}{\sum_{i=1}^{n} K_s^O} \tag{8}$$

$$EF = \frac{\sum_{i=1}^{N}(O_i - \bar{O})^2 - \sum_{i=1}^{N}(P - O_i)^2}{\sum_{i=1}^{N}(O_i - \bar{O})^2} \tag{9}$$

$$\text{ME} = \max \left| P_i - O_i \right|_{i=1}^{n} \tag{10}$$

where O_i and P_i represent measured and estimated saturated hydraulic conductivity by means of the models, respectively, and N indicates the number of soil samples.
The minimum value for ME and RMSE is zero. Both EF and CRM can be negative. The EF value compares the estimated values to the averaged measured values. A negative EF value indicates that the

averaged measured values give a better estimate than the estimated values. The maximum value for EF is one. A large ME and RMSE value represent the worst performance of the evaluated model. The RMSE quantifies the dispersion of the measured and estimated values with respect to the 1:1 line. Smaller value of RMSE indicates smaller deviation, or higher between the values estimated and measured. The CRM is a measure of the tendency of the model to overestimate or underestimate the measurements. A negative CRM indicates a tendency to overestimation. If all estimated and measured data are the same, these statistics values: ME = 0; RMSE = 0; EF = 0 and CRM = 0 (Zarei *et al*., 2011; Ghorbani *et al*., 2010).
The accuracy and the reliability of the models were also evaluated using the statistic Pearson correlation coefficient r. The Pearson correlation coefficient indicates a measure of linear relationships between the measured and estimated data. The r ranges from −1 (a perfect negative relationship) to + 1 (a perfect positive relationship). The larger absolute value of r indicates stronger relationships. The r can be obtained from:

$$r = \frac{\left(n\sum_{i=1}^{n}(P_i)(O_i)\right)-\left(\sum_{i=1}^{n}P_i\right)\left(\sum_{i=1}^{n}O_i\right)}{\sqrt{n\left[\sum_{i=1}^{n}(P_i)^2-\left(\sum_{i=1}^{n}(P_i)\right)^2\right]\left[n\sum_{i=1}^{n}(O_i)^2-\left(\sum_{i=1}^{n}O_i\right)^2\right]}}, \qquad -1 \le r \le +1 \tag{11}$$

where O_i and P_i represent measured and estimated saturated hydraulic conductivity by means of the models, respectively, and n indicates the number of soil samples.

RESULTS AND DISCUSSION

The grain-size distribution curve of some selected soil samples from UNSODA are drawn in Figure (1). The grain-size distribution is theoretically a continuous curve representing the amount various partial size present in soil. The
vertical axis is analogous to probability.

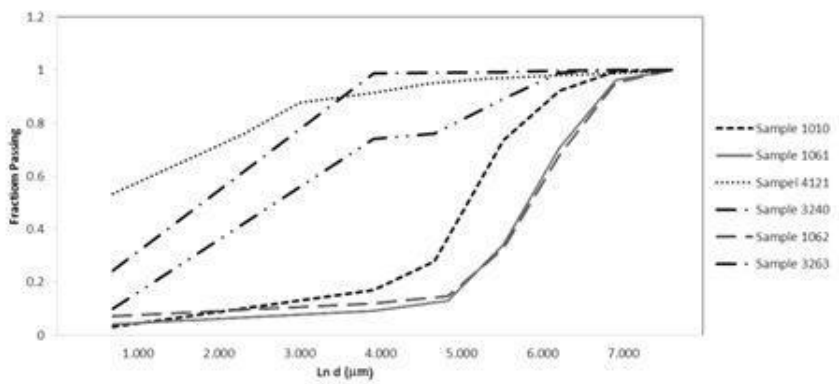

Figure 1: Grain- size distribution curves for the four samples used in the application section

It should be noted that they both overlap and overlay the range of soil grain sizes (Table 1 and 2). For triangular fuzzy sets the left and right parameters are the upper and lower bounds of the fuzzy set with zero membership and the middle value is the median (full membership), while trapezoidal fuzzy sets differ by having two median parameters that define an interval of values with full membership.

Table 1:Fuzzy form of soil grain types

Soil grain type	Fuzzy numbers (μm)
Clay	[1, 1, 2, 26]
Silt	[2, 26, 500]
Sand	[26, 500, 1025, 2000]

Table 2: Fuzzy form of soil saturated hydraulic conductivity fuzzy sets

Linguistic term	fuzzy numbers (cm/day)

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Low	[-1, -0.35,0.35, 1]
Medium	[0.35, 1.3, 1.6, 2.5]
High	[1.6, 2.5,3.3 ,4.4]

The PD of the selected soil samples is depicted in Figure (3). The soil sample 1062 with sand 88%, silt 7% and clay 5% intersect the sand fuzzy (Figure 2). However, the soil sample 2362 with 21%, silt 16% and clay 63% intersect clay fuzzy set (Figure 3). The soil sample with more sand particles intersects the sand fuzzy set at maximum membership function. But in soil sample 2362 with the less sand particles, the PD of the soil sample intersects clay fuzzy set at maximum point. Therefore, the particles percentage would determine the maximum intersection point. If the PD of samples intersect fuzzy sets at several points, the maximum point of intersection at each fuzzy set would be important to calculate.

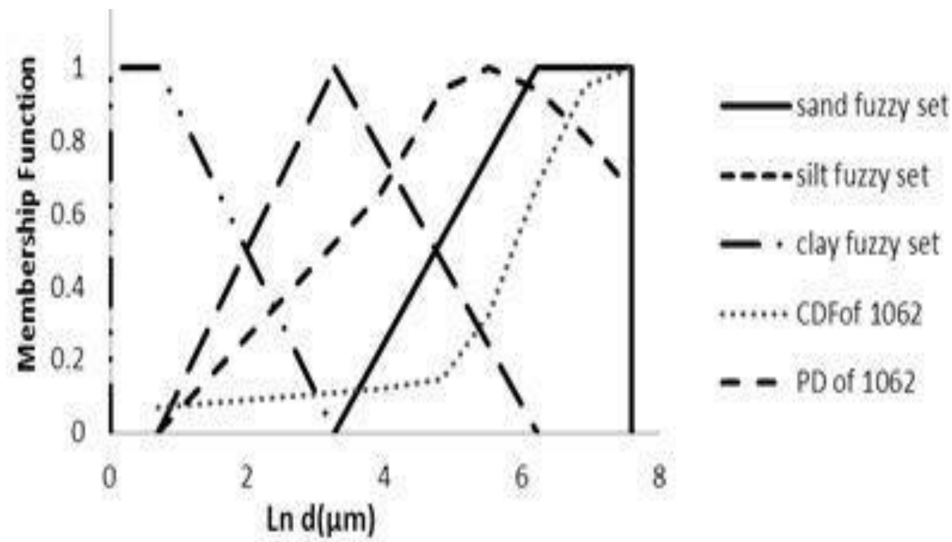

Figure 2: Possibility distributions (PD) of the soil sample 1062

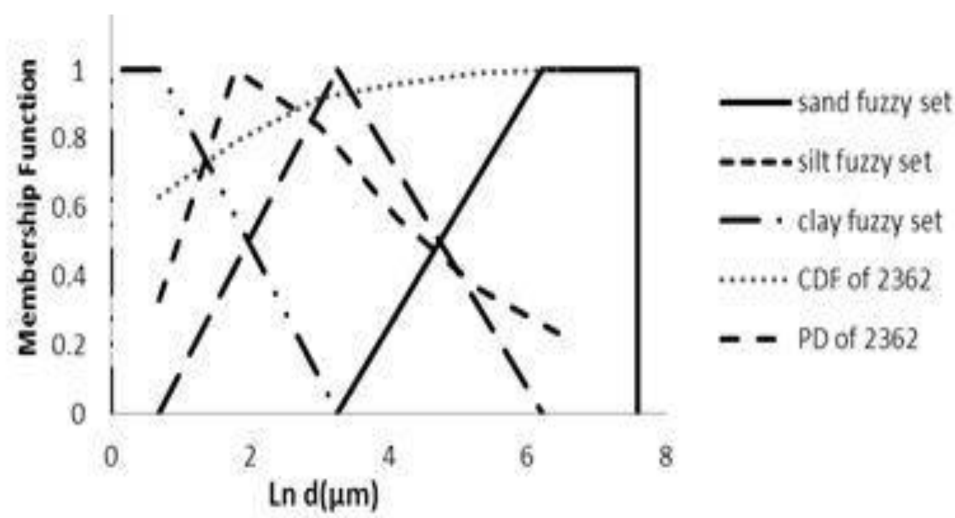

Figure 3: Possibility distributions (PD) of the soil sample 2362

Because, the maximum point of intersection between the possibility distribution and each fuzzy set is the degree that activates the rules in MFIS. The MFIS that consists of three rules of a membership function for each estimated variables to estimate saturated hydraulic conductivity (log (cm/day)). The each of these inputs activated the rules and resulted in a set of three fuzzy hydraulic conductivity output values (Figure 4). Three classes with membership functions for each input variable were generated and with the fuzzy rules of the Mamdani inference engine in MFIS. The membership of all classes were aggregated and defuzzified to obtain an estimated log Ks.

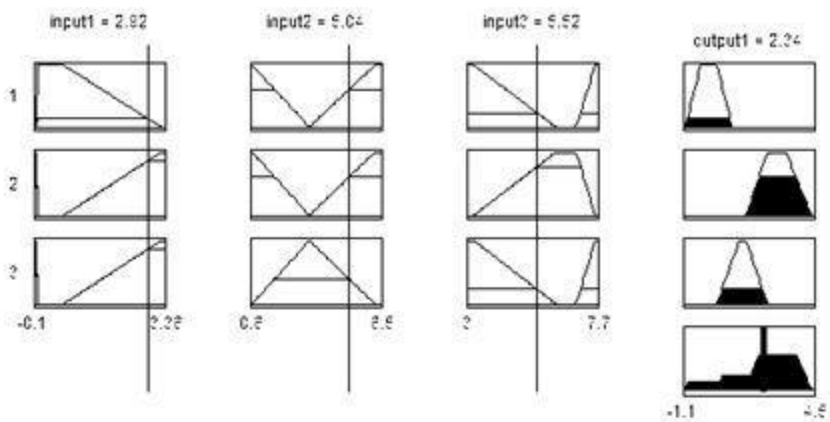

Figure 4: MFIS with three rules for sample of 2105

Moreover, K_s was estimated by the point PTFs. The derived point PTFs represented in Table 2 with I, II and IV respectively. Table 2 shows the derived point PTFs of soil samples with clay content (C), sand content (S) and σ_g as input parameters with their RMSE and $R^2{}_{adj}$. The adjusted coefficients of determination ($R^2{}_{adj}$) indicate the variation range of the independent variable that was explained by the dependent variables.

Table 3: The derived point PTFs that predict K_s using C and S percentages and σ_g as input parameters

	The derived pedotransfer functions	$R^2{}_{adj}$ (RMSE)
I	$K_s = \exp(-0.799 \ln C + 3.583)$	0.53 (0.75)
II	$K_s = \exp\left(-11.675\,(\log \sigma_g)^{\frac{1}{4}} + 13.087\right)$	0.522 (1.048)
III	$K_s = \exp(-0.524 \ln S + 3.021)$	0.239

It is obvious that the smaller the RMSE values, and the more *r* values, the better performance will be achieved for the method. In the follow, the PTFs were ranked according to the least absolute values of RMSE and the highest value of R^2.
The functions given in Table 3 indicate that the use of clay content and σ_g instead of sand, contents can better estimate K_s. Table (3) provides a quantitative comparison of the saturated hydraulic conductivity estimated with the fuzzy approach, the derived point PTFs and Rosetta package. In practically all cases, the point PTFs (II) had the largest RMSE values and was therefore the least accurate estimator, so that this model has lower performance. Based on the obtained RMSE performance of the Rosetta package is better than PTFs (II). Compared with fuzzy approach, the accuracy of the point PTFs was not good. Our results indicate that the point PTFs could not accurately predict the saturated hydraulic conductivity. Compared with PTFs and Rosetta package, the fuzzy approach did lead to better accuracy with RMSE= 0.67. Also, we found that the Rosetta package provides better estimates of K_s than PTFs (II). A large ME value represents the worst case performance of the model. The negative CRM indicate a tendency to

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overestimation of all methods except for PTFs (II). The PTFs (II) introduced the highest bias (consistent under or overestimation), followed by PTFs (II), Rosetta package, fuzzy approach. The fuzzy approach yielded the lowest bias. Figures 4, 5 and 6 show measured the measured K_s values versus K_s estimated with fuzzy approach, PTFs and Rosetta package. In all, the results indicate relatively good performance of the fuzzy approach in terms of the three statistics (Table 4).

Table 4: The calculated statistics for evaluating the accuracy of fuzzy approach, point PTFs and Rosetta package

Methods	RMSE	r	ME	CRM	EF
Fuzzy approach	0.67	0.91	1.467	-0.3	0.506
PTFs (I)	0.75	0.75	2.71	0.17	0.434
PTFs (II)	1.048	0.49	2.9988	-0.67	0.212
ROSETTA+SSC	0.823	0.82	2.21	-0.35	0.6
ROSETTA+SSC+BD	0.793	0.83	2.48	-0.36	0.62

Also, the reliability of the prediction is evaluated by the correspondence between measured and predicted values (Wosten *et al*., 2001).
The estimated values by fuzzy approach, PTFs and Rosetta and their measured values were also plotted along the 1:1 line for a visual view of their performance (Figures 5,6 and 7).
Figures 5, 6 and 7 indicate that fuzzy approach provide better estimate of K_s in comparison with derived PTFs and Rosetta package.

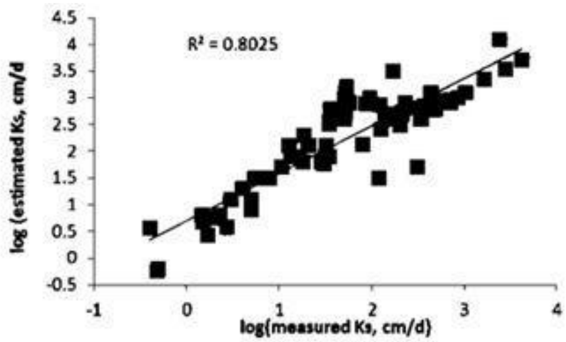

Figure 5: Measured K_s vs. K_s values predicted with fuzzy approach

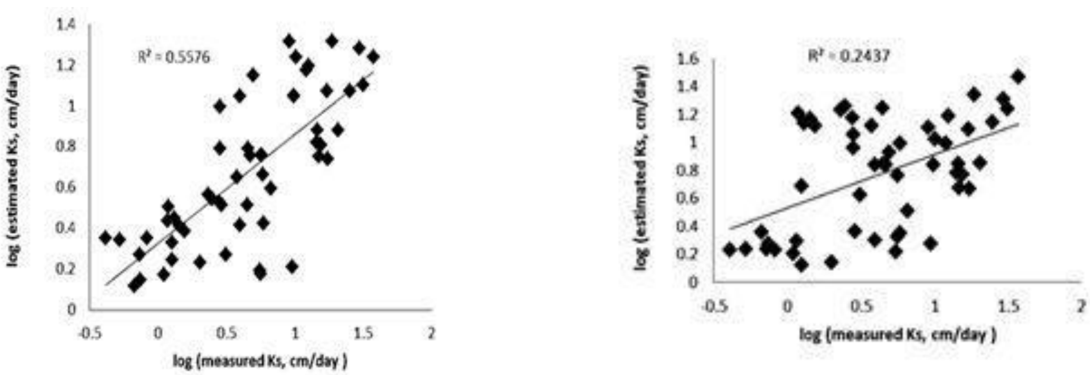

Figure 6: The measured K_s vs. K_s values predicted with derived PTFs. by Eq.(I) (left- hand)and Eq. (II) (right-hand)

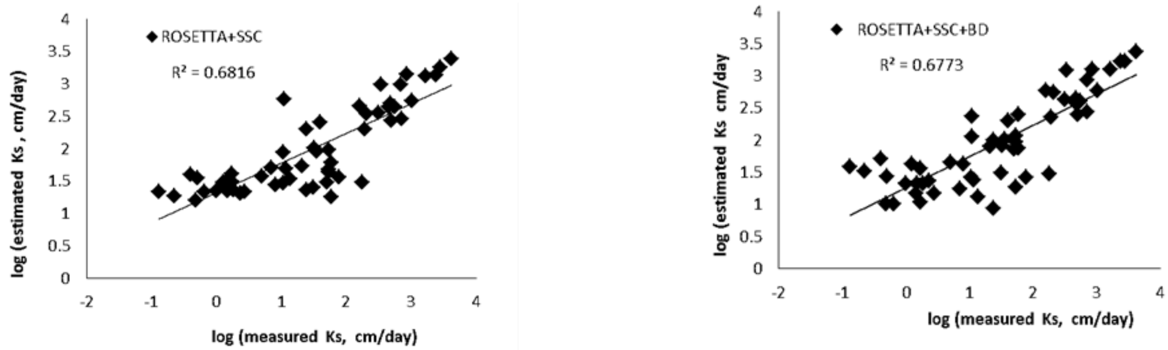

Figure 7: Measured K_s vs. K_s values predicted with Rosetta

Conclusion

In this paper, an alternative approach, based on fuzzy set theory and fuzzy rule-based system, has been introduced to estimate K_s by using PSD as input variables. Also, our results indicated that fuzzy approach is very effective in predicting K_s in comparison with derived PTFs and Rosetta. Compared with PTFs and Rosetta, the fuzzy approach provided better accuracy, with RMSE not exceeding 0.823 cm/day. The results show that the fuzzy approach provides the highest accuracy, followed by Rosetta package and PTFs. Several scientists introduced many methods based on fuzzy set theory such as Ross *et al*., (2007). The major advantage in comparison with Ross *et al*., is that the K_s of a large number of soil samples can be estimated by fuzzy approach in this paper. Therefore, it allowed us to evaluate the accuracy and efficiency of the proposed model with statistics defined in previous sections. Estimating K_s with multiple inputs in a short time is another advantage of the proposed method in comparison with Ross *et al*` method. Therefore, in addition PSD, the other soil properties such as soil structure and effective porosity can be used as inputs to estimate K_s. Also, the proposed method is entirely based on fuzzy mathematical models. The results indicate that the proposed method allowed us to estimate K_s with acceptable accuracy. Furthermore, fuzzy approach allowed us to incorporate both hard data (measured) and soft data (expert judgment) for expressing inherent variability in input variables. However, the fuzzy approach still needs to be improved by considering effects of soil structure and soil hydraulic properties. Therefore, further studies are needed to incorporate fuzzy set theory to groundwater flow and transport models.

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