NATURE OF GENERALISED ALGEBRAIC STRUCTURE
\[ A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 / a_i \in F & G_i \in C(P) = \text{CLASS OF ALGEBRAIC STRUCTURE}\} \]

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ABSTRACT
This is my sincere efforts towards realization of Unchanging Truth. This work is dedicated to my spiritual teacher Sri Sri Ramakrishna. In the Present work first I proved that \((A, +, \cdot)\) is a non abelian ring. Second I proved \((A, +, \ast)\) is a commutative ring with unity, where \(A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 / a_i \in F & G_i \in C(P)\), and \(C(P) = \text{Class of algebraic Structure}\).

Keywords: Binary Operation, Abelian Group, Ring, Field, Class of Algebraic Structure

INTRODUCTION
Herstein cotes in 1992

Definition: A nonempty set of elements \(G\) is said to form a group if in \(G\) there is defined a binary operation, called the product and defined by \(*\), such that
1 a, b \(\in G\) implies that \(a*b \in G\)
2 a, b, c \(\in G\) implies that \((a*b)*c = a* (b*c)\)
3 There exist an element \(e \in G\) such that \(a*e = e*a = a\) for all \(a \in G\)
4 For every \(a \in G\) there exist an element \(a^{-1} \in G\) such that \(a* a^{-1} = a^{-1} * a = e\)

Definition: A group \(G\) is said to be abelian (or Commutative) if for every \(a, b \in G\),
\[ a * b = b * a. \]

Definition: A nonempty set \(R\) is said to be an associative ring if in \(R\) there are defined two operations, defined by + and \(*\) respectively, such that for all \(a, b, c\) in \(R\):
1 \(a+b\) is in \(R\).
2 \(a+b = b+a\).
3 \((a+b)+c = a+(b+c)\).
4 There is an element \(0\) in \(R\) such that \(a+0 = a, \forall a \in R\)
5 There exist an element \(-a\) in \(R\) such that \(a + (-a) = 0\).
6 \(a*b\) is in \(R\)
7 \(a*(b*c) = (a*b)*c\).
8 \(a * (b+c) = a * b + a * c\) and \((b+c) * a = b*a + c*a\).

It may very well happen, or not happen, that there is an element \(1\) in \(R\) such that \(a*1 = 1*a = a\) for every \(a\) in \(R\); if there is such we shall describe \(R\) as a ring with unit element.

If the multiplication of \(R\) is such that \(a*b = b*a\) for every \(a, b\) in \(R\), then we call \(R\) a commutative ring.

DISCUSSION
Let \(A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 / a_i \in F & G_i \in C(P)\}\)
Where \(C(P) = \text{Class of algebraic Structure}\)
Let \(x = a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3;\)
\(y = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3;\)
\(z = c_0G_0 + c_1G_1 + c_2G_2 + c_3G_3;\)
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\[ G_0 = 1G_0 + 0G_1 + 0G_2 + 0G_3 \\
0 = 0G_0 + 0G_1 + 0G_2 + 0G_3 \]

\[-x = (-a_0)G_0 + (-a_1)G_1 + (-a_2)G_2 + (-a_3)G_3 \]

\[ cx = (ca_0)G_0 + (ca_1)G_1 + (ca_2)G_2 + (ca_3)G_3, \quad c \in F \]

Here first binary operation + on A defined as
\[ x + y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3) + (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3) \]
\[ = (a_0 + b_0)G_0 + (a_1 + b_1)G_1 + (a_2 + b_2)G_2 + (a_3 + b_3)G_3 \]

\[ \cdots \cdots (1) \]

\[ => x + y = y + x, \quad \forall x, y \in A \]
\[ x.(y + z) = (x + y) + z, \quad \forall x, y, z \in A \]
\[ 0 + x = x + 0, \quad \forall x \in A \]

\[ x + (-x) = (-x) + x = 0, \forall x \in A \]

\[ \Rightarrow (A,+) \text{ is an abelian group.} \cdots \cdots (2) \]

**Case 1:**

**Second binary operation. on A defined as**
\[ x, y = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3), \quad (b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3) \]
\[ = (a_0 + a_1 + a_2 + a_3) y, \quad \forall x, y \in A \cdots \cdots (3) \]
\[ x, y \neq y, x, \quad \forall x, y, z \in A \]
\[ x.(y.z) = (x.y).z, \forall x, y, z \in A \]

Hence \((A,\cdot)\) is a non abelian semi group \cdots \cdots (4)

Also
\[ x.(y + z) = x.y + x.z, \forall x, y, z \in A \]
\[ (x + y).z = x.y + y.z, \forall x, y, z \in A \]

From (1) to (5), one obtains
\( (A,+,\cdot) \) is a non abelian ring.

**Case 3:**

**Second binary operation \(*\) on A defined as**
\[ x \ast y = (a_0b_0 + a_1b_3 + a_2b_2 + a_3b_1)G_0 + (a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2)G_1 \]
\[ + (a_0b_2 + a_1b_1 + a_2b_0 + a_3b_3)G_2 + (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0)G_3 \]

\[ \cdots \cdots (6) \]

\[ => x \ast y = y \ast x, \forall x, y \in A \]
\[ x \ast (y \ast z) = (x \ast y) \ast z, \forall x, y, z \in A \]

\[ G_0 \ast x = x \ast G_0 = x, \forall x \in A \]

Let \(x^{-1} = b_0G_0 + b_1G_1 + b_2G_2 + b_3G_3\) be the inverse of \(x = (a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3)\) in A.

\[ \Rightarrow \text{ By definition we get} \]
\[ x \ast x^{-1} = x^{-1} \ast x = G_0 \]

\[ (a_0b_0 + a_1b_3 + a_2b_2 + a_3b_1) = 1 \cdots \cdots (7) \]
\[ (a_0b_1 + a_1b_0 + a_2b_3 + a_3b_2) = 0 \cdots \cdots (8) \]
\[ (a_0b_2 + a_1b_1 + a_2b_0 + a_3b_3) = 0 \cdots \cdots (9) \]
\[ (a_0b_3 + a_1b_2 + a_2b_1 + a_3b_0) = 0 \cdots \cdots (10) \]

Rewriting eq\(^{(8)}\), \((9)\) \&(10), one obtains as

\[ (a_1b_0 + a_0b_1 + a_3b_2 + a_2b_3) = 0 \]
\[ (a_2b_0 + a_1b_1 + a_0b_2 + a_3b_3) = 0 \]
\[ (a_3b_0 + a_2b_1 + a_1b_2 + a_0b_3) = 0 \]

\[ \Rightarrow \frac{b_0}{F_0} = \frac{-b_1}{F_1} = \frac{b_2}{F_2} = \frac{-b_3}{F_3} = k \text{ (constant).} \cdots \cdots (11) \]
\[
F_0 = \begin{bmatrix}
a_0 & a_3 & a_2 \\
a_1 & a_0 & a_3 \\
a_2 & a_1 & a_0 \\
a_3 & a_2 & a_1
\end{bmatrix}
\]
\[
F_1 = \begin{bmatrix}
a_0 & a_3 & a_2 \\
a_1 & a_0 & a_3 \\
a_2 & a_1 & a_0 \\
a_3 & a_2 & a_1
\end{bmatrix}
\]
\[
F_2 = \begin{bmatrix}
a_0 & a_3 & a_2 \\
a_1 & a_0 & a_3 \\
a_2 & a_1 & a_0 \\
a_3 & a_2 & a_1
\end{bmatrix}
\]
\[
b_0 = kF_0,
\]
\[
b_1 = -kF_1,
\]
\[
b_2 = kF_2,
\]
\[
b_3 = -kF_3
\]
\[
b_0 = k(a_0^3 - 2a_0a_1a_3 + a_2(a_1^2 + a_3^2) - a_0a_2^2)
\]
\[
\text{……….. (12)}
\]
\[
b_1 = -k[a_1^3 - 2a_0a_2a_3 + a_1(a_0^2 + a_2^2) - a_3a_1^2]
\]
\[
\text{……….. (13)}
\]
\[
b_2 = \{a_1^3 - 2a_1a_2a_3 + a_0(a_1^2 + a_3^2) - a_2a_0^2\}
\]
\[
\text{……….. (14)}
\]
\[
b_3 = -k[a_1^3 - 2a_0a_1a_2 + a_3(a_0^2 + a_2^2) - a_1a_3^2]
\]
\[
\text{……….. (15)}
\]
\[
=> k = \frac{1}{\{a_0^4 - a_1^4 + a_2^4 - a_3^4\} + 4a_0a_2(a_1^2 + a_3^2) - 4a_1a_3(a_0^2 + a_2^2) - 2a_0^2a_2^2 + 2a_1^2a_3^2}
\]
\[
= \infty \text{ if } a_0 = a_1 & a_2 = a_3
\]
Hence \(x^1\) of each \(x \in A\) does not exist.

\(\rightarrow (A, \ast)\) is a commutative monoid \(\text{………..(16)}\)

also \(x \ast (y + z) = x \ast y + x \ast z, \forall x, y, z \in A\)
\(x \ast (y + z) = x \ast z + y \ast z, \forall x, y, z \in A\)
\(\text{……….. (17)}\)

\((A, +, \ast)\) is a commutative ring with unity.
Where \(A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \mid a_i \in F \& G_i \in C(P)\}\)
And \(C(P) = \text{Class of algebraic Structure}\)

\textbf{Conclusion}
From the above discussion, I come to the following conclusions-
first I proved that \((A, +, \ast)\) is a non abelian ring. Second I proved \((A, +, \ast)\) is a commutative ring with unity.
Where \(A = \{a_0G_0 + a_1G_1 + a_2G_2 + a_3G_3 \mid a_i \in F \& G_i \in C(P)\}\) and \(C(P) = \text{Class of algebraic Structure}\).

\textbf{REFERENCES}