RADIATION EFFECT ON AN UNSTEADY MHD FLOW PAST A POROUS PLATE WITH HALL CURRENT IN A ROTATING SYSTEM

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ABSTRACT
An exact solution to the problem of an unsteady free convective flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate in a rotating system taking into account the effect of Hall current is presented when the temperature as well as the concentration at the plate varies periodically with time. The flow is in presence of appreciable radiation and uniform transverse magnetic field and the fluid rotates with a constant angular velocity about the normal to the plate. The Magnetic Reynolds number is considered small enough to neglect the induced hydromagnetic effects. The expressions for the temperature, concentration, and velocity field, skin friction at the plate, the Nusselt number and Sherwood number are obtained in non-dimensional form. Detailed computations of the influence of Hartmann number, radiation parameter and Hall parameter on the variations in these fields are demonstrated graphically and physically interpreted.

Keywords: MHD, Hall Current, Radiation

INTRODUCTION
Many natural phenomena and technological problems are susceptible to MHD analysis. Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields, engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. The study of MHD is quite important in the field of missile technology, aerodynamics since the temperature that occurs in such flight speeds are sufficient to dissociate or ionize the air appreciably and the motion of this ionized air may be controlled by applying a magnetic field suitably. The study of MHD is also relevant in medical science. For instance there have been researches on Arteriosclerosis (the cause of a cardiac arrest) where the effect of externally applied transverse magnetic field on a pulsatile flow in constricted arteries (tubes) is considered. When an electrically conducting fluid flows past a flat plate, its motion can be retarded by applying a transverse magnetic field and the Lorentz force acts as a resistance force in the direction opposite to the direction of the fluid velocity. Due to this the skin friction at the plate is reduced and hence the boundary flow may be controlled by transverse magnetic field.

The geophysical importance of the flows in rotating frame of reference has attracted the attention of a number of scholars. Investigation of the combined effects of rotation and magnetic field on MHD flow has been an active topic of research because of its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering. MHD in its present form is due to the pioneer contribution of several notable authors like Alfven (1942), Shercliff (1965), Ferraro and Plumpton (1966) and Crammer and Pai (1978). It was emphasized by Cowling (1957) that when the strength of the magnetic field is sufficiently large, Ohm’s law needs to be modified to include Hall current. Hall effects are significant when the density of the fluid is low and/ or the applied magnetic field is strong. It plays a vital role in determining flow features of the fluid flow problems. It is significant to study the combined effects of Hall current and rotation on MHD flow problems. Taking into consideration this fact, Ahmed and Kalita (2011), Sattar and Maleque (2000), investigated this fluid flow problem considering different aspects.

The natural flow arises in fluid when the temperature change causes density variation leading to buoyancy forces acting on the fluid. Free convection is a process of heat transfer in natural flow. The heating of rooms and buildings by use of radiator is an example of heat transfer by free convection. Radiation is another process of heat transfer through electromagnetic waves. Radiative convective flows are
encountered in countless industrial and environment processes like heating and cooling chambers, evaporation from large open water reservoirs, astrophysical flows and solar power technology. Due to importance of the above physical aspects, several authors have carried out model studies on the problems of free convective flows of incompressible viscous fluid under different flow geometries taking into account of the thermal radiation. Some of them are Mansour (1990), Raptis and Perdikis (1999), Mbeledogu et al. (2007), Makinde (2005) and Sattar and Kalim (1996), Samad and Rahman (2006), Orhan and Ahmet (2008), Prasad et al. (2006), and Ahmed (2012).

In this proposed work, the applied magnetic field, radiation and Hall current are taken into account in order to investigate the effects on the flow and transport characteristics.

Mathematical formulation

The equations governing the motion of an incompressible, viscous, electrically conducting radiating fluid in a rotating system in presence of a magnetic field are:

Equation of continuity:

\[ \nabla \cdot \mathbf{q} = 0 \]  \hspace{1cm} (2.1)

Momentum equation:

\[ \rho \left( \frac{\partial \mathbf{q}}{\partial t} + 2\tilde{\Omega} \times \mathbf{q} + \tilde{\Omega} \times (\tilde{\Omega} \times \mathbf{r}) + (\mathbf{q} \cdot \nabla) \mathbf{q} \right) = -\nabla \mathbf{p} + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} + \mu \nabla^2 \mathbf{q} \]  \hspace{1cm} (2.2)

Energy equation:

\[ \rho C_p \left( \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T \right) = k \nabla^2 T + \Phi + \frac{\mathbf{J}^2}{\sigma} - \frac{\mathbf{\partial q} \cdot \mathbf{\partial y}}{\partial y} \]  \hspace{1cm} (2.3)

Species continuity equation:

\[ \frac{\partial \mathbf{C}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{C} = D \nabla^2 \mathbf{q} \]  \hspace{1cm} (2.4)

Kirchhoff’s first law:

\[ \nabla \cdot \mathbf{J} = 0 \]  \hspace{1cm} (2.5)

General Ohm’s law:

\[ \frac{\alpha \tau}{B_0} \mathbf{J} + \mathbf{J} \times \mathbf{B} = \sigma \left[ \nabla \times \mathbf{E} + \frac{1}{\epsilon_0} \nabla \mathbf{p}_e \right] \]  \hspace{1cm} (2.6)

Gauss’s law of magnetism:

\[ \nabla \cdot \mathbf{B} = 0 \]  \hspace{1cm} (2.7)

We now consider the unsteady flow of a viscous incompressible electrically conducting fluid. The flow occurs over an infinite vertical porous plate in a rotating system with constant suction taking into account the Hall current and radiation in presence of a uniform transverse magnetic field. Our investigation is restricted to the following assumptions:

1. All the fluid properties except the density in the buoyancy force term are constants.
2. The plate is electrically non-conducting.
3. The entire system is rotating with angular velocity \( \tilde{\Omega} \) about the normal to the plate.
4. The magnetic Reynolds number is so small that the induced magnetic field can be neglected. Also the electrical conductivity of the fluid is reasonably low and hence the Ohmic dissipation may be neglected.
5. The electron pressure \( p_e \) is constant.
6. \( \mathbf{E} = 0 \) i.e. the electric field is negligible.
7. \( |\tilde{\Omega}| \) is so small that \( |\tilde{\Omega} \times (\tilde{\Omega} \times \mathbf{r})| \) i.e. the centrifugal force may be neglected.
We introduce a coordinate system \((x, y, z)\) with \(x\)-axis is oriented vertically upwards along the plate and \(y\)-axis is taken normal to the plane of the plate and \(z\)-axis along the width of the plate as shown in figure 1. The plate is subjected to a constant suction velocity \(v_0\).

Let \(\vec{q} = \hat{u} + \hat{v} + \hat{w}\) be the fluid velocity, \(\vec{J} = J_x \hat{i} + J_y \hat{j} + J_z \hat{k}\) be the current density at the point at the point \(P(x, y, z, t)\) and \(\vec{B} = B_0 \hat{j}\) be the applied magnetic field.

**Figure 1: Physical model of the problem**

The equation (2.5) gives \(\frac{\partial \vec{J}_y}{\partial y} = 0\) which shows that \(\vec{J}_y = \text{constant}\). Since the plate is electrically non-conducting, \(\vec{J}_y \bigg|_{y=0} = 0\) and is zero everywhere in the flow and consequently the current density is given by \(\vec{J} = J_x \hat{i} + J_z \hat{k}\).

The assumptions V and VI lead the equation (2.6) to the following form

\[
\vec{J} + \frac{\omega_x \tau_e}{B_0} \left( \vec{J} \times \vec{B} \right) = \sigma \left( \vec{q} \times \vec{B} \right)
\]

where \(m = \omega_x \tau_e\) is the Hall parameter.

Equations (2.9) yield

\[
\begin{align*}
J_x &= \frac{\sigma B_0}{1 + m^2} (m\vec{u} - \vec{w}) , \\
J_z &= \frac{\sigma B_0}{1 + m^2} (\vec{u} + m\vec{w}) ,
\end{align*}
\]

Since the plate is infinite in extent all physical quantities are the function of \(y\) and \(\tau\) only. With the
foregoing assumptions and under the usual boundary layer and Boussinesq approximation the equations (2.1) to (2.4) reduce to the following set of equations:

\[
\frac{\partial \bar{v}}{\partial y} = 0 \quad \text{which yields} \quad \bar{v} = -v_0 \tag{2.12}
\]

\[
\frac{\partial \bar{u}}{\partial t} + \bar{v} \frac{\partial \bar{u}}{\partial y} + 2\bar{\Omega} \bar{w} = \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma B_0^2}{\rho (1 + m^2)} \left( \bar{u} + mw \right) + g\beta \left( \bar{T} - \bar{T}_\infty \right) + g\beta \left( \bar{C} - \bar{C}_\infty \right) \tag{2.13}
\]

\[
\frac{\partial \bar{w}}{\partial t} + \bar{v} \frac{\partial \bar{w}}{\partial y} - 2\bar{\Omega} \bar{u} = \nu \frac{\partial^2 \bar{w}}{\partial y^2} - \frac{\sigma B_0^2}{\rho (1 + m^2)} \left( \bar{m}u - \bar{w} \right) \tag{2.14}
\]

\[
\frac{\partial \bar{T}}{\partial t} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial \bar{q}_t}{\partial y} \tag{2.15}
\]

\[
\frac{\partial \bar{C}}{\partial t} + \bar{v} \frac{\partial \bar{C}}{\partial y} = \frac{D}{\rho C_p} \frac{\partial^2 \bar{C}}{\partial y^2} \tag{2.16}
\]

Where, \( \frac{\partial \bar{q}_t}{\partial y} = 4I (T' - T'_\infty) \)

The appropriate boundary conditions are

\[
y = 0: \quad \bar{u} = 0, \quad \bar{w} = 0, \quad \bar{T} = \bar{T}_\infty + \left( \bar{T}_w - \bar{T}_\infty \right) e^{i\omega t}, \quad \bar{C} = \bar{C}_\infty + \left( \bar{C}_w - \bar{C}_\infty \right) e^{i\omega t} \tag{2.17a}
\]

\[
y \to \infty: \quad \bar{u} \to 0, \quad \bar{w} \to 0, \quad \bar{T} \to \bar{T}_\infty, \quad \bar{C} \to \bar{C}_\infty \tag{2.17b}
\]

We now introduce the following non-dimensional quantities:

\[
y = \frac{v_0}{v} \bar{y}, \quad t = \frac{v_0^2 \bar{t}}{v}, \quad u = \frac{\bar{u}}{v_0}, \quad w = \frac{\bar{w}}{v_0}, \quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_0 - \bar{T}_\infty}, \quad \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_0 - \bar{C}_\infty}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}, \quad Gr = \frac{g\beta v}{v_0^3} \left( \bar{T}_0 - \bar{T}_\infty \right), \quad Gm = \frac{g\beta v}{v_0^3} \left( \bar{C}_0 - \bar{C}_\infty \right), \quad M = \frac{\sigma v B_0^2}{\rho v_0^2}, \tag{2.18}
\]

\[
\Omega = \frac{2\bar{\Omega} v}{v_0^2}, \quad \eta = \frac{\nu \bar{v}}{v_0^2}, \quad Q = \frac{4I v}{\rho C_p v_0^2} \tag{2.19}
\]

The dimensionless forms of the equations (2.13) to (2.16) are

\[
\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \Omega = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{M}{1 + m^2} (u + mw) + Gr\theta + Gm\phi \tag{2.20}
\]

\[
\frac{\partial \bar{w}}{\partial \bar{t}} - \frac{\partial \bar{w}}{\partial \bar{y}} - \bar{w} \Omega = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{M}{1 + m^2} (mu - w) \tag{2.21}
\]

\[
\frac{\partial \bar{\theta}}{\partial \bar{t}} - \frac{\partial \bar{\theta}}{\partial \bar{y}} = \frac{1}{Pr} \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} - Q\bar{\theta} \tag{2.22a}
\]

\[
\frac{\partial \bar{\phi}}{\partial \bar{t}} - \frac{\partial \bar{\phi}}{\partial \bar{y}} = \frac{1}{Sc} \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} \tag{2.22b}
\]


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\[ y \to \infty : u = 0, w = 0, \theta = 0, \phi = 0 \]  \hspace{1cm} (2.22b)

**Method of solution**

Introducing the complex variable \( q = u + iw, \quad i = \sqrt{-1} \), the equations (2.18) and (2.19) transform to single partial differential equation

\[ \frac{\partial q}{\partial t} + \frac{\partial^2 q}{\partial y^2} \left[ \frac{M}{1 + m^2} (1 - im) - i\Omega \right] q + Gr\theta + Gm\phi = 0 \]  \hspace{1cm} (3.1)

The corresponding boundary conditions become

\[ y = 0: q = 0, \theta = e^{in}, \phi = e^{in} \]  \hspace{1cm} (3.2a)

\[ y \to \infty : q = 0, \theta = 0, \phi = 0 \]  \hspace{1cm} (3.2b)

In order to solve the equations (2.20), (2.21) and (3.1) under the boundary conditions (2.22), we assume

\[ q = q_0 (y) e^{in}, \theta = \theta_0 (y) e^{in}, \phi = \phi_0 (y) e^{in} \]  \hspace{1cm} (3.3)

Substituting (3.3) into the equations, we obtain

\[ q_0'' (y) + q_0' (y) - (A_1 + i\eta) q_0 (y) = -Gr\theta_0 (y) - Gm\phi_0 (y) \]  \hspace{1cm} (3.4)

\[ \theta_0'' (y) + Pr \theta_0' (y) - (i\eta Pr + Q Pr) \theta_0 (y) = 0 \]  \hspace{1cm} (3.5)

\[ \phi_0'' (y) + Sc\phi_0' (y) - i\eta Sc\phi_0 (y) = 0 \]  \hspace{1cm} (3.6)

The corresponding boundary conditions reduce to

\[ y = 0: q_0 = 0, \theta = 1, \phi = 1 \]  \hspace{1cm} (3.7a)

\[ y \to \infty : q_0 = 0, \theta = 0, \phi = 0 \]  \hspace{1cm} (3.7b)

The solutions of the equations (3.4), (3.5) and (3.6) subject to the boundary conditions are as follows:

\[ q_0 (y) = A_2 e^{-\lambda y} + A_3 e^{-\lambda y} + A_4 e^{-\lambda y} \]  \hspace{1cm} (3.8)

\[ \theta_0 (y) = e^{-\lambda y} \]  \hspace{1cm} (3.9)

\[ \phi_0 (y) = e^{-\lambda y} \]  \hspace{1cm} (3.10)

Hence the non dimensional velocity, temperature and concentration distributions are given by

\[ q = A_2 e^{in-\lambda y} + A_3 e^{in-\lambda y} + A_4 e^{in-\lambda y} \]  \hspace{1cm} (3.11)

\[ \theta = e^{in-\lambda y} \]  \hspace{1cm} (3.12)

\[ \phi = e^{in-\lambda y} \]  \hspace{1cm} (3.13)

Splitting (3.11) into real and imaginary parts, we obtain

\[ u = A_{11} e^{-Xy} + A_{12} e^{-Xy} + A_{13} e^{-Xy} \]  \hspace{1cm} (3.14)

\[ w = B_{11} e^{-Xy} + B_{12} e^{-Xy} + B_{13} e^{-Xy} \]  \hspace{1cm} (3.15)

Separating (3.12) and (3.13) into real and imaginary parts, the real part is given by

\[ \theta = e^{-Xy} \cos (\eta t - Y_2y) \]  \hspace{1cm} (3.16)

\[ \phi = e^{-Xy} \cos (\eta t - Y_4y) \]  \hspace{1cm} (3.17)

**Skin-friction:**

The axial component of the skin friction at the plate is

\[ \tau_x = \left( \frac{\partial u}{\partial y} \right)_{y=0} = \tau_{x_1} \cos \eta t + \tau_{x_2} \sin \eta t = |\tau_x| \cos (\eta t + \alpha) \]  \hspace{1cm} (4.1)
The transverse component of the shearing stress at the plate is
\[
\boldsymbol{\tau}_y = \frac{\partial \mathbf{w}}{\partial y} = \tau_{z_2} \cos \omega t + \tau_{z_1} \sin \omega t = |\tau_y| \cos (\eta t + \beta)
\]  
(4.2)

where
\[
|\tau_y| = \sqrt{\tau_{x_2}^2 + \tau_{z_1}^2}
\]
\[
\alpha = \tan^{-1} \left( \frac{\tau_{z_1}}{\tau_{x_2}} \right) \quad \beta = \tan^{-1} \left( \frac{\tau_{z_1}}{\tau_{x_2}} \right)
\]
\[
\tau_{x_2} = -(X_0 Y_2 + X_2 Y_9 + X_{11} Y_4 + X_4 Y_{11} + X_{12} Y_7 + X_7 Y_{12})
\]
\[
\tau_{z_1} = (Y_0 Y_2 - X_2 Y_9 + X_4 Y_{11} - X_{11} X_4 + Y_{12} Y_7 - X_{12} Y_7)
\]

**Rate of heat transfer:**

The heat flux from the plate to the fluid in terms of Nusselt number \( \text{Nu} \) is given by
\[
\text{Nu} \frac{\partial \theta}{\partial y} = -X_2 \cos \eta t + Y_2 \sin \eta t = |\text{Nu}| \cos (\eta t + \gamma)
\]
\[
|\text{Nu}| = \sqrt{X_2^2 + Y_2^2} \quad \gamma = \tan^{-1} \left( \frac{Y_2}{X_2} \right)
\]

**Rate of mass transfer:**

The mass flux from the plate to the fluid in terms of Sherwood number \( \text{Sh} \) is given by
\[
\text{Sh} \frac{\partial \phi}{\partial y} = -X_4 \cos \eta t + Y_4 \sin \eta t = |\text{Sh}| \cos (\eta t + \delta)
\]
\[
|\text{Sh}| = \sqrt{X_4^2 + Y_4^2} \quad \delta = \tan^{-1} \left( \frac{Y_4}{X_4} \right)
\]

The constants \( \lambda_1, \lambda_2, \lambda_3, A_1, A_2, A_3, A_4, A_{11}, A_{12}, A_{13}, B_{11}, B_{12}, B_{13}, X_1, ..., X_{12}, Y_1, ..., Y_{12} \) are not shown here for the sake of brevity.

**RESULTS AND DISCUSSION**

In order to get clear insight of the physical problem, numerical computations from the analytical solutions for the representative temperature field, concentration field, velocity field, the co-efficient of skin friction, the rate of heat transfer at the plate in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number have been carried out by assigning some arbitrarily chosen specific values to the physical parameters involved in the problem, the normal coordinate \( y \) and time \( t \). Throughout our investigation, the value of \( \text{Pr} \) have been kept fixed at 0.71, both the values of \( \text{Gr} \) and \( \text{Gm} \) are fixed at 5 and 2 respectively as the numerical computations are concerned. We recall that \( \text{Pr} = 0.71 \) corresponds physically to air. The numerical results computed from the analytical solutions of the problem have been displayed in figure 2-7.

Figures 2 and 3 present how the fluid velocity is affected by radiation and applied magnetic field respectively. These two figures uniquely show that an increase in the radiation parameter and Hartmann number results in a steady decrease in the fluid velocity thereby reducing the thickness of the velocity boundary layer. These two figures further reveal that the fluid velocity first increases in a thin layer adjacent to the plate and there after it decreases asymptotically as we move away from the plate indicating
the fact that the buoyancy force has a significant effect on the flow near the plate and its effect is nullified in the free stream.

**Figure 2:** Velocity versus \( y \) under the effect of radiation

**Figure 3:** Velocity versus \( y \) under the effect of applied magnetic field

Figure 4 corresponds to the temperature distribution \( \theta \) against \( y \) under the influence of \( Q \) shows an opposing influence on \( \theta \) indicating the fact that the fluid temperature falls steadily for high radiation. It is
further noticed from these figures that as expected the fluid temperature asymptotically falls from its maximum value at \( y = 0 \) to its minimum value at \( y \to \infty \).

\[
\theta
\]

\[
Q=1, Q=2, Q=3
\]

\[
y \to \infty
\]

\[\Rightarrow\]

**Figure 4: Temperature versus \( y \) under the effect of radiation**

Figures 5 and 6 demonstrate the effect of the Hartmann number \( M \) and Radiation parameter \( Q \) on the skin friction at the plate. Both the figures indicate that the magnitude of shear stress at the plate is considerably decreased with the increase in \( M \) and \( Q \). The effect of Hall parameter is immaterial on the axial component of the skin friction at the plate whereas the magnitude of the transverse component rises for increasing values of Hall parameter.

\[
\tau_x, \tau_y
\]

\[
Q=1, Q=2, Q=3
\]

**Figure 5: Shearing Stress versus Hall parameter under the effect of radiation**
The effect of the radiation parameter on the co-efficient of the rate of heat transfer in terms of the Nusselt number Nu have been displayed in figure 7. This figure predicts that magnitude of Nu is constantly decreased for increasing values of radiation parameter.
Conclusions
Our investigation may be summarized to the following conclusions:
i) The imposition of the transverse magnetic field retards the flow. As a consequence of this, the growth of thickness of the velocity boundary layer is prevented to some extent which in turn stabilizes the flow.
ii) High radiation causes the fluid temperature to fall and thereby reduces the thickness of the thermal boundary layer.
iii) Magnitude of shear stress at the plate is considerably decreased due to application of transverse magnetic field.
iv) High radiation leads the substantial fall in the heat transfer rate.

Nomenclature
\( \vec{B} \) is the Magnetic induction vector,
\( B_0 \) is the strength of the applied magnetic field,
\( C_p \) is the Specific heat at constant pressure,
\( \bar{C} \) is the species concentration,
\( D \) is the coefficients of mass diffusivity,
\( \vec{E} \) is the electric field,
\( \vec{g} \) is the gravitational acceleration vector,
\( Gr \) is the Grashof number for Heat transfer,
\( Gm \) is the Grashof number for Mass transfer,
\( \vec{J} \) is the current density vector,
\( k \) is the thermal conductivity,
\( M \) is the Hartmann number,
\( Pr \) is the Prandtl number,
\( p \) is the pressure,
\( \vec{q} \) is the fluid velocity vector,
\( Q \) is the radiation parameter,
\( q_r \) is the radiative heat flux,
\( t \) is the time,
\( \rho \) is the fluid density,
\( \mu \) is the co-efficient of viscosity,
\( \sigma \) is the electrical conductivity,
\( \Phi \) is the viscous dissipation of energy per unit volume,
\( \beta \) is the co-efficient of volume expansion for heat transfer,
\( \beta_\theta \) is the co-efficient of volume expansion for mass transfer,
\( \theta \) is the non-dimensional temperature,
\( \phi \) is the non dimensional concentration,

ACKNOWLEDGEMENT
The author is highly thankful to Prof. N. Ahmed, Professor, Department of Mathematics, Gauhati University for his valuable discussion and suggestions in this research work.

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