MODIFIED GAIN CHARACTERISTICS OF LONGITUDINAL ELECTRO-KINETIC WAVES IN COLLOIDS LADEN QUANTUM SEMICONDUCTOR PLASMAS

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ABSTRACT
Gain characteristics of longitudinal electro-kinetic wave (LEKW) in multi-component (electron, ion and colloid) quantum plasmas are studied. A compact linear dispersion relation for electro-kinetic waves in colloids laden quantum semiconductor plasmas has been derived by using quantum hydrodynamic (QHD) model. This dispersion relation has been studied for slow and fast electro-kinetic modes. It is found that the quantum Bohm potential term modifies the gain spectra of all the four modes of propagation under slow and fast wave limit in multi-component colloids laden quantum semiconductor plasmas.

Key Words: Electrostatic Waves, Plasma Effects, Colloidal Plasma, Bohm Potential

INTRODUCTION
Substantial progress has been made in the study of elementary wave excitations in gaseous plasma and their interactions with the constituents of the systems. The gaseous plasmas may contain some additional micron/sub-micron sized fine solid particles, which increases the complexity of the system, hence termed as complex/dusty plasma. If these foreign components are charged by some process inside the plasma, it introduces some unique structure and alters the properties and spectra of various waves supported by it. A significant interest in the field of complex plasma is generated and has become one of the active research frontiers in the last decade (Salimullah et al., 2004).

During the last decade, the dusty plasma concept has reoriented the direction of research in semiconductor plasmas also (Salimullah et al., 2000; Salimullah et al., 2003). When ions are implanted inside the semiconductor material they agglomerated to form colloids. The presence of these colloids in addition to mobile charged carriers converts the medium to the multi-component semiconductor plasma medium. It is known fact that the study of wave propagation through a medium always provide information about the properties of the host material, therefore such ion-implanted semiconductor that resembles dusty plasma system become promising medium to study wave propagation phenomena during the last decade (Ghosh and Thakur, 2004; 2005; 2006; Ghosh et al., 2006; Ghosh and Khare, 2005; 2006(a); 2006(b)).

If the de-Broglie wavelength of charge carriers becomes comparable to the dimension of the system, the quantum mechanical effects are supposed to play a lead role in deciding the nature of the wave spectra. The plasma physicists have attracted towards quantum plasma (Roy and Chatterjee, 2011; Akbari, 2012) because of its manifold applications from nano-science (Craighead, 2000) to astrophysics (Shapiro and Teukolsky, 2007).

Out of the many theoretical models the quantum hydrodynamic model (QHD) becomes popular to study the wave phenomena in quantum plasma system. Due to the mathematical convenience in studying the microscopic quantities of particles and achieving analytical results, the QHD model has been extensively utilized in resonant tunneling diode, super fluid and superconductivity (Feynman, 1972; Gardener and Ringhofer, 1996; Coste, 1998). As our knowledge goes there are only very few reports on the wave characteristics in colloids laden quantum semiconductor plasmas, hence we decided to study probably for the first time the gain characteristic of longitudinal electro-kinetic waves (LEKW) in colloids laden quantum semiconductor plasma using QHD model. It was expected that the inclusion of Bohm potential term through QHD model in the momentum equation would modify the wave spectra in multi-component colloids laden quantum semiconductor plasmas.
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Theoretical Formulation

Here we have considered the QHD model which is an extension of the classical fluid model of plasma. It is comprised of a set of equations that describe the transport of momentum and energy of charge species. The departure from the classical model lies in the fact that an additional term, called “Bohm potential” is introduced in equation of motion of the charged particles. In the limit that the quantum effects go to zero the classical fluid equation of motion is retrieved in accordance with the corresponding principle.

Using QHD model, we have derived the linear dispersion relation for LEKW in colloids laden semiconductor quantum plasma and analyzed under different frequency limits. For this we have considered n-type colloids laden semiconductor crystal of infinite extent. The condition for charge neutrality in this system is given by

$$z_an_{oh} = z_en_{0e} + z_hn_{0h}$$  \hspace{1cm} (1)

where \( n_{0\alpha} (\alpha = e, h, d) \) are unperturbed number densities of electrons, holes and colloids, \( z_{\alpha} \) is the charge states of carriers, in which \( z_{\alpha} = (q_{d} / e) \) is the ratio of negative charges \( q_{d} \) resided over the colloidal grains to the charge \( e \) on electrons.

In multi-component plasma system, if \( m_{\alpha}, \ z_{\alpha}, \ n_{\alpha}, \ \text{and} \ v_{\alpha} \) are the mass, charge state, density, and momentum transfer collision frequency of each component, respectively then the continuity and momentum equations are given by

$$\frac{\partial n_{\alpha}}{\partial t} + \frac{\partial}{\partial z} (n_{\alpha} v_{\alpha}) = 0$$  \hspace{1cm} (2)

$$\frac{\partial v_{z\alpha}}{\partial t} = \frac{z_{\alpha} q_{\alpha}}{m_{\alpha}} E_{z} - v_{\alpha} v_{z\alpha} - \frac{1}{m_{\alpha} n_{0\alpha}} \left[ \nabla P_{\alpha} - \frac{\hbar^2 \nabla^2 (n_{2\alpha})}{4m_{\alpha}} \right]$$  \hspace{1cm} (3)

where, \( P_{\alpha} = \frac{m_{\alpha} v_{F_{\alpha}}^2 n_{\alpha}^3}{3n_{0\alpha}^2} \) is the Fermi pressure with \( v_{F_{\alpha}} = \left( \frac{2k_{B}T_{e}}{m_{\alpha}} \right) \) being the Fermi speed.

Following the standard procedure and by assuming the variation of the wave as \( \exp[i(\omega - k \cdot z)] \), we obtain the general dispersion relation for longitudinal electro-kinetic wave in colloids laden quantum semiconductor plasma from equations (2) and (3) as

$$\varepsilon(\omega, k) = 1 + \left( \frac{\omega_{pe}^2}{\omega^2 - i \nu_{e}\omega - k^2 V_{Fe}^2 (1 + \gamma_{e})} \right) + \left( \frac{\omega_{ph}^2}{\omega^2 - i \nu_{h}\omega - k^2 V_{Fh}^2 (1 + \gamma_{h})} \right) + \frac{\omega_{pd}^2}{\omega^2} = 0$$  \hspace{1cm} (4)

where, \( \omega_{pe} = \frac{(z_e e)^2 n_{0e}}{e m_{e}}, \ \gamma_{e} = \frac{\hbar^2 k^2}{e m_{e} T_{e}}, \ \text{and} \ \varepsilon = \varepsilon_{0} \varepsilon_{L}; \ \varepsilon_{L} \) being the lattice dielectric constant.

On neglecting the quantum correction \( (\gamma_{\alpha} = 0) \) term, the dispersion relation given by equation (4) reduces to dispersion relation obtained by (Ghosh and Thakur, 2004). Hence the derived dispersion relation for LEKW shows the modifications due to quantum effect in colloid laden semiconductor plasmas through \( \gamma_{\alpha} \) and \( V_{F\alpha} \) terms.

We shall study this relation under different frequency regimes to achieve some idea about the modification made by the quantum effect in gain characteristics of electro-kinetic mode propagation through colloids laden quantum semiconductor plasma.

Slow electro-kinetic wave (SEKW) \((\omega << V_{Fe}, kV_{Fh})\)

If the phase velocity of the wave is less than the Fermi velocities of electrons and holes both, the mode may be termed as slow electro-kinetic mode. Therefore, for slow electro-kinetic mode (SEKM), under collision dominated or low frequency regime \((\omega << V_{Fe}, V_{Fh})\), dispersion relation equation (4) reduces to

$$1 - \frac{\omega_{pe}^2}{(i \nu_{e} + k^2 V_{Fe}^2 (1 + \gamma_{e}))} - \frac{\omega_{ph}^2}{(i \nu_{h} + k^2 V_{Fh}^2 (1 + \gamma_{h}))} + \frac{\omega_{pd}^2}{\omega^2} = 0$$  \hspace{1cm} (5)
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Equation (5) may be written in the form of polynomial in \( \omega \) as,

\[
\omega^4 - i\omega^3 \left[ \omega_{re} \left\{ k^2 \tilde{\omega}_{pe}^2 \left( 1 + \gamma_e \right) - 1 \right\} + \omega_{hm} \left\{ k^2 \tilde{\omega}_{ph}^2 \left( 1 + \gamma_h \right) - 1 \right\} \right] \\
- \omega^2 \left[ k^2 \left\{ k^2 \tilde{\omega}_{pe}^2 \left( 1 + \gamma_e \right) - \tilde{\omega}_{ph}^2 \left( 1 + \gamma_h \right) - \tilde{\omega}_{hm}^2 \left( 1 + \gamma_h \right) \right\} \right] \\
- i\omega \left[ k^2 \tilde{\omega}_{ph}^2 \tilde{\omega}_{hm} \omega_{re} \left( 1 + \gamma_h \right) \right] = 0
\]

(6)

where, \( \omega_{re,ph} = \frac{\omega_{re,ph}}{v_{e,h}} \) are the dielectric relaxation frequencies of electrons and holes, respectively.

Fast electro-kinetic wave (FEKW) \((kV_{ph} << \omega << kV_{pe})\)

If the phase velocity of the mode is less than electron Fermi velocity but more than the hole Fermi velocity, the mode may be termed as fast electro-kinetic mode (FEKM). Thus for fast electro-kinetic mode, the dispersion relation equation (4) reduces to,

\[
1 - \frac{\omega_{pe}^2}{(iv_{e}\omega + k^2V_{re}^2(1+\gamma_e))} + \frac{\omega_{ph}^2}{(\omega^2 - iv_{e}\omega)} + \frac{\omega_{pd}^2}{\omega^2} = 0
\]

(7)

Equation (7) may be rewritten in the term of polynomial in \( \omega \) as,

\[
\omega^4 \omega_{ph} - i\omega^3 \left[ \tilde{\omega}_{ph}^2 + \left\{ k^2 \tilde{\omega}_{pe}^2 \left( 1 + \gamma_e \right) - 1 \right\} \omega_{re} \omega_{ph} \right] \\
- \omega^2 \left[ k^2 \tilde{\omega}_{pe}^2 \left( 1 + \gamma_e \right) - 1 \right] \omega_{re} \omega_{ph}^2 - \omega_{ph} \omega_{re} \left( \omega_{ph}^2 + \omega_{pd}^2 \right) \right] \\
- i\omega \left[ \tilde{\omega}_{ph} \omega_{ph} \omega_{pd} + k^2 \tilde{\omega}_{pe} \omega_{re} \omega_{ph} \left( 1 + \gamma_e \right) \right] \left( \omega_{ph}^2 + \omega_{pd}^2 \right) = 0
\]

(8)

For both the frequency regimes, the obtained dispersion relations are polynomials of fourth degree in terms of angular frequency \( \omega \). Hence we shall have to solve them numerically to study the gain characteristics of LEKW in ion-implanted quantum semiconductor plasmas.

RESULTS AND DISCUSSION

We have considered the perturbation of the wave as \( \exp \left( i \omega (t - kz) \right) \), thus the mode would be growing in time when \( \omega_i < 0 \) and decaying when \( \omega_i > 0 \). For numerical estimations of gain profile for both kinds of modes of LEKW in presence of quantum correction terms, we shall analyze the equations (6) and (7) derived for colloids laden quantum semiconductor plasmas. For the same, the following representative parameters have been used: \( m_e = 0.0815m_0, m_0 \) being the free electron mass, \( m_h = 4m_e \), \( m_d = 10^{-27} \) kg, \( \varepsilon_L = 15.8 \), \( n_{be} = 10^{20} \) m\(^{-3}\), \( n_{bh} = 3 \times 10^{20} \) m\(^{-3}\), \( n_{bd} = 10^{24} \) m\(^{-3}\), \( \nu_e = 3.463 \times 10^{13} \) sec\(^{-1}\), \( \nu_h = 1.194 \times 10^{11} \) sec\(^{-1}\) and \( \nu_d = 3.422 \times 10^{10} \) sec\(^{-1}\).

It is found that only the III- mode of propagation out of four possible modes in SEKW limit is found to be growing \( (\omega_i < 0) \) in nature. The gain profile of all the four modes of SEKW in presence and absence of quantum effect are depicted in Figures1-4.
Figures 1 and 2 display the gain profiles of I- and II-mode respectively with respect to positive wave number $k$ in presence and absence of quantum effect. It may be inferred from these two figures that both the modes are decaying in nature in the wavelength limit under study. Even the presence of quantum effect does not change the nature of gain profile. The attenuation coefficients increase with increment in $k$. The quantum correction is found to be responsible for more attenuation. The difference in attenuations also increases with $k$. Hence it may be concluded that the presence of Bohm potential in colloids laden semiconductor plasma is responsible for enhancement in attenuations of I-and II-mode in case of SEKW.
The variation of growth rate $\omega_i$ ($\omega_i < 0$) in presence and absence of quantum effect for III-mode of SEKW with wave number $k$ is depicted in Figure 3. The growth rate of this mode, first decreases very rapidly with $k$ up to $k \leq 3 \times 10^7 \text{ m}^{-1}$. In this regime of $k$ the quantum effect modifies the growth rate significantly. The presence of Bohm potential decreases the magnitude of growth rate effectively in this regime of wave number. Further if we increase $k$ beyond this limit ($k > 3 \times 10^7 \text{ m}^{-1}$) the growth rate becomes nearly independent of $k$ and found identical in both the cases (with and without quantum effect). Hence at higher $k$ or at lower wavelength the quantum effect becomes negligible for III-mode of SEKW.

Figure 3: Variation of gain coefficient of III-mode of SEKW with wave number $k$

Figure 4: Variation of gain coefficient of IV-mode of SEKW with wave number $k$
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The decaying gain profile for IV-mode of SEKW is illustrated in Figure 4. It is found that the gain profile of IV-mode is just opposite of III-mode as depicted in Figure 3. From Figure 4, it may be infer that attenuation rates first decrease rapidly with increase in value of k up to k=3×10^{7} m^{-1}, and for k>3×10^{7} m^{-1} it becomes independent of it. Figure also depicts that the presence of quantum effect in colloid laden semiconductor plasma modifies the attenuation coefficient of this mode only in the higher wavelength regime. Similarly the gain profiles of all the possible propagating modes (I-III) of FEKW in presence and absence of quantum effect are depicted in Figures 5-7. The nature of gain profile of IV- mode of FEKW is found to be attenuating in nature in the wavelength limit under study. Its variation with wave number k is found to be very insignificant, hence graphical illustration is not given.

![Figure 5: Variation of gain coefficient of I-mode of FEKW with wave number k](image)

The gain profile of I-mode of FEKW with respect to k is depicted in Figure 5. This counter propagating mode (\omega_r<0) is found to be decaying (\omega_i>0) in nature always. This figure also depicts that the quantum correction term is found to be responsible for enhancement of attenuation coefficient of I-mode of FEKW. This rate of enhancement of attenuation coefficient increases with k.

![Figure 6: Variation of gain coefficient of II-mode of FEKW with wave number k](image)
The attenuation profile of II-mode of FEKW with wave number k in presence and absence of quantum effect is depicted in Figure 6. It is found that this mode is also decaying in nature for both the cases (with and without quantum effect). It is also found that its attenuation rate first decrease with increase in the value of k and then becomes independent of k at higher values. Quantum correction reduces the magnitude of attenuation coefficient effectively towards higher wavelength regime.

![Figure 6: Attenuation profile of II-mode of FEKW](image)

**Figure 7: Variation of gain coefficient of III-mode of FEKW with wave number k**

The profile of III-mode of FEKW is illustrated in Figure 7. In FEKW limit this is the only mode of propagation which is found to be amplifying in nature in both the cases (with and without quantum effect). The growth rate of this mode, first decreases very rapidly with k up to $k \leq 3 \times 10^7 \text{ m}^{-1}$, in this regime of k the quantum effect modifies the growth rate significantly. Further if we increase k beyond this limit ($k > 3 \times 10^7 \text{ m}^{-1}$) the growth rate becomes nearly independent of k in both the cases (with and without quantum effect).

**Conclusion**

It is found that the presence of quantum correction terms change the gain profile only quantitatively in all the cases. It does not affect the gain profile qualitatively. Even though it may be concluded from above study that the inclusion of Bohm potential in hydrodynamic model of colloid laden semiconductor plasma is very essential to understand the gain profiles of all the four modes of propagations of LEKW under slow and fast wave limit.

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