Trochanteric Fracture – Its Managements Established by Mathematical Devices

*Swapan Kumar Adhikari
Netaji Subhas Open University
35/I, Krishnataran Naskar Lane, Ghusuri, Howrah, West Bengal – 711 107
*Author for correspondence

ABSTRACT
Viability of use of cross-screw in addition to DHS (Dynamic Hip Screw), for fixing inter-trochanteric fracture [Fig.1B] at upper part of femur [Fig.1A] to resist the torsion due to rotational forces exerted by ligament and muscles during rest and movement (Adhikari and Saha, 2011), has been established. The method has been proved to be by mathematical deduction.

Key-Words: Dynamic hip screw, shearing force, shearing stress, torsion-stress, trochanteric-fracture.

INTRODUCTION
Here it will be appropriated to quote the words of Gathorne Robert Girdlestone, Oxford Orthopedic Surgeon, expressed in 1932: “Human body is that where bone is a plant, with its roots in soft tissue. When its vascular connections are damaged, it often requires, not only the techniques of a cabinet maker, but also the patients’ care as understanding of a guardian”.

According to insertion of interdisciplinary approach in treating a fracture every orthopedic surgeons are very much careful and chalk out a plan as an architect. Before going through the operation to regain the normalcy, surgeons undertake following subjects (a) Vascular supply; (b) Muscle-tendon unit damage; (c) skin coverage; (d) degree of soft tissue damage; (e) amount of neurological damage; (f) microbiology of environment; (g) viability of nutritional support.

In search of literature on the development of intertrochanteric fracture management, starting from consideration of sliding compression screw, I find following dissertations: (a) Doppelt (1980) conducted retrospective study to compare efficacy of sliding compression screw and the Jewett nail in stabilizing intertrochanteric fractures and recorded 2.9% failure in sliding hip screw whereas 40.8% failure in Jewett nail. Ultimately he claimed that “Sliding compression screw is today’s best answer for stabilization of intertrochanteric hip fractures”. (b) Jensen (1980) also demonstrated that “In comminuted and unstable intertrochanteric fractures sliding hip screw was the most suitable implant”. (c) Chang (1987) reported that “An anatomical reduction of four fragment intertrochanteric fracture internally fixed with a sliding compression hip screw is able to provide higher compression across calcar region and project lower tensile force on plate”. (d) Bannister et al (1990) find that “The dynamic hip screw was three times effective than Jewett nail plate in controlling collapse of the intertrochanteric fractures”. So, he recommended the use of dynamic hip screw in treatment of trochanteric fractures as it is best accommodated. (e) Rao et al (1990) had taken the cases to compare compression hip screw fixation and enders nail in treatment of intertrochanteric fractures of hip. Finally they preferred compression hip screw as the treatment of choice in both stable and unstable intertrochanteric fractures of femur. (f) O’Brien et al (1995) compared the cases of using of gamma nail and dynamic hip screw for fixation of intertrochanteric fractures. In this study they commented that “Dynamic hip screw should be considered as implant of intertrochanteric fractures for its low risk in complication”. (g) Gundle et al (1995) declared that “Sliding hip screw has improved the treatment of trochanteric fractures due to sharing of load between the implant and the fracture fragments. (h) Bartle et al (1996) implanted anti-rotational parallel screw with dynamic hip screw fixation. (i) Adams et al (2001) stated that “Use of an intramedullary device in the treatment of trochanteric fractures of the femur is associated with higher but non-significant risk of post-operative complications. Routine use of gamma nail cannot be recommended over the current standard treatment of DHS and plate”. (Fig. 2 & 3). (j) Sommer et al., (2004) tried to establish the cut out resistance of implants used in fixation of trochanteric fractures and they found that DHS failed under cyclic loading. (k) Edward et al (2003) concluded that “Attachment of additional lateral support to a sliding hip screw significantly decrease displacement of the femoral head after cyclical loading”.

43
After review of pertinent literature sources concerning hip fracture epidemiology, hip fracture injury mechanisms, and hip fracture management strategies, it has been revealed that hip fractures have several causes, but among these, the impact of falls and muscle weakness, along with low physical activity levels seems to be the most likely to be the rising incidence of hip fracture injuries. Related determinants of suboptimal nutrition, drugs that increase fall risk and lower the safety threshold and co-morbid conditions of the neuromuscular system may also contribute to hip fracture disability (Marks et al., 2003; Hesse 2004). Neither age related osteoporosis nor the increasing incidence of falls with age sufficiently explains the exponential increase in the incidence of hip fracture with aging. We propose that four conditions must be satisfied in order for a fall to cause a hip fracture: (a) the fatter must be oriented to impact near the hip; (b) protective responses must fail; (c) local soft tissues must absorb less energy than necessary to prevent fracture, and (d) the residual energy of the fall applied to the proximal femur must exceed its strength. All of these events become more likely with aging and lead to an exponential rise in the risk of hip fracture with advancing age. This model also suggests that a combination of measurements of neuromuscular...
function and of bone strength may be the most accurate approach to assessing the risk of hip fracture. According to the epidemiologic projections on hip fractures worldwide annual number will rise to 6.26 million by the year 2050. This rise will be in great part due to the huge increase in the elderly population of the world. However, the age-specific incidence rates of hip fractures have also increased during the recent decades and in many countries. Reasons for the age-specific increase are not known: increase in the age-adjusted incidence of falls of the elderly individuals with accompanying deterioration in the age-adjusted bone quality (strength, mineral density) may partially explain the phenomenon. The growth of the elderly population will be more marked in Asia, Latin America, the Middle East, and Africa than in Europe and North America, and it is in the former regions that the greatest increments in hip fracture are projected so that these regions will account for over 70% of the 6.26 million hip fractures in the year 2050. The incidence rates of hip fractures vary considerably from population to population and race to race but increase exponentially with age in every group [Kannu (1996)].

A number of interventions may help to prevent hip fracture injuries, including, interventions that optimize bone mass and quality, interventions that help prevent falls and falls dampening interventions. Rehabilitation outcomes may be improved by comprehensive interventions, prolonged follow-up strategies and ensuring that all aging adults enjoy optimal health.

We know that all weight bearing bones are of internally trabecular pattern [Fig.4], having bone marrow centrally, to resist mechanical stresses and strains. It is a viscoelastic, biphasic substance analogous to fiber-glass containing tropocollagen to its fiber and crystalline hydroapatite to the glass (Adhikari 2003).

**METHODS**

The DHS method was introduced with an idea that at the time of fixation of trochanteric fractures cannot be fitted face to face or by snug-fitting. So, for a better possibility of correction at the time of walking or weight bearing, this it was taken to be the best device [Bannister et al, 1990]. But in the modern times, fixations of such fractures are done by the use of TV-monitor and other devices [Appollo Hospitals & Bhattacharyya Orthopaedic and Related Research Centre]. We know that bone-structure or calcar femorale in the trochanteric region of femur is dense. Therefore, flexibility is merely needed after fixation. DHS has to be given for absorbing greater amount of force on the basis of bending etc as because pipe can absorb more force than a rod of same diameter. Compression [Fig.5] is the weight / force which transmitted to plate by DHS. Also this force is resisted and absorbed by the bone due to its compactness of helical trabecules. Here the compression-force on the head i.e. on the upper end of DHS is accompanied by a parallel force $C_1$ at the lower end of DHS. This forms a couple. This couple is attached with the plate and $C_1 < C$ as a part of the force C is being absorbed by the bone itself due flexibility and weight bearing capacity of bone. $C_1$ cannot be much intensified by the weight C of the upper part of the body that it can uproot the screws attached with the shaft; it cannot even extent the plate. The plate is in extensible. So bending of head against DHS is not possible by this force.
Translation [Fig. 6] force is generally of lower intensity than compression. It is not enough to bend the plate because most of the angular force P on the head of the femur is coming from the upper part of body as weight and angle $\theta$ is less than $45^o$, so $C > T$ and if $\theta > 45^o$ then leg is above the ground i.e. in the hanging position and all the weight of the body is transferred to the other leg and free leg is used to balance the upper part of the body. The only force which can deform the DHS in the management of Trochanteric Fracture is twisting force. This force is effective enough to twist the plate (Fig.7). The DHS plate is thus that the longitudinal dimension ‘a’ of this plate is much more than the thickness ‘t’ of the plate. Screw parallel to DHS cannot resist the twist as it is in one direction i.e. the system employed in twodimensional. But the twisting force is three-dimensional. So, twisting force can only be resisted by the threedimensional management.
The twisting forces are very often on the femur due to walking and movement of femur. Muscles and ligaments attached with the femur transmit forces from different angles. Even very small twisting of plate is enough for deforming the attachment of DHS with parallel screw. The net effect is lowering of the head of the femur.

The cross screw used with DHS (Fig.8) has been used for three-dimensional management of trochanteric fracture. Up till now a nail was passed through the medullar cavity attached with DHS and screwed on the shaft of the femur to resist the torsion force [Fig.9]. This method can only change moment of the weight which is very small but cannot bear the total brunt effectively. This acts the head of the femur by shifting the fulcrum from the end to the middle of DHS. This system may also interfere with the normal functions of bone-marrow.

Dr. (Prof) Sailendra Bhattacharyya first used a cross screw in place of parallel screw to DHS. This proved to be effective.
This idea of using cross screw with DHS can be evaluated by following mathematical deductions [Xu, (1993)].

Let us consider an elementary part of the plate as parallelepiped [Fig.10] with dimension \( dx \), \( dy \), and \( t \). Stress components perpendicular to x-axis are \( \sigma_x \), \( \tau_{xy} \), \( \tau_{xz} \), where \( \sigma_x = \) Bending stress, \( \tau_{xy} = \) Torsion stress, \( \tau_{xz} = \) Transverse shearing stress. \( \sigma_y \propto z \), where \( z \) is the distance of point of action of the stress from the middle of the plate.

So, Bending Moment per unit width of the plate is

\[
M_x = \int \left( \sigma_x \cdot 1 \cdot dz \right) = \int \frac{1}{2} \cdot z \cdot \sigma_x \cdot dz
\]

But \( \sigma_x = -\left( \frac{Ez}{1+\mu^2} \right) \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \)

where \( E = \) Modulus of Elasticity or Young’s Modulus of the plate = \( \frac{\text{Stress}}{\text{Strain}} \) [Stress = Force per unit area]

\( \mu = \) Poisson’s ratio = \( \frac{\text{Longitudinal strain}}{\text{Longitudinal strain}} \)

\( \omega = \) deflection and it is a function of \( x \) and \( y \) where \( \frac{\partial^2 \omega}{\partial x^2}, \frac{\partial^2 \omega}{\partial y^2} \) are second order partial differentiation of \( \omega \) with respect to \( x \) and \( y \) respectively

So,

\[
M_x = -\frac{E}{1-\mu^2} \int \frac{1}{2} \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \cdot z \cdot dz = -\frac{E \cdot t^2}{12(1-\mu^2)} \int \frac{1}{2} \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \cdot z^2 \cdot dz
\]

Similarly, torsion stress \( \tau_{xy} \propto z \), then twisting moment per unit width of the plate is

\[
M_{xy} = \int \frac{1}{2} \cdot z \cdot \tau_{xy} \cdot dz = -\frac{E}{1+\mu} \int \frac{1}{2} \cdot z \cdot \frac{\partial^2 \omega}{\partial x \partial y} \cdot z^2 \cdot dz
\]

Transverse shearing stresses per unit width of the plate are \( \tau_{xz}, \tau_{xy} \) and \( \tau_{xz} = \tau_{xy} \).

So, Transverse Shearing Stress moment is

\[
Q_x = \int \tau_{xz} \cdot dz = -\frac{E}{2(1-\mu^2)} \int \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \cdot z^2 \cdot \left( \frac{z^2 - t^2}{4} \right) \cdot dz
\]

\[
= -\frac{E \cdot t^2}{12(1-\mu^2)} \int \frac{1}{2} \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \cdot \left( z^2 - \frac{t^2}{4} \right) \cdot \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \cdot \frac{1}{2} \cdot z^2 \cdot dz
\]

In the same way Stress components on the normal to the y-axis are \( \sigma_y, \tau_{yz}, \) and \( \tau_{yz} \) which are symbolic as the same way before and respective moments of twisting and shearing forces are

\[
M_y = \int \frac{1}{2} \cdot z \cdot \sigma_y \cdot dz = -\frac{E \cdot t^2}{12(1-\mu^2)} \int \frac{1}{2} \cdot z \cdot \frac{\partial^2 \omega}{\partial y^2} \cdot \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial x^2} \right) \cdot \frac{1}{2} \cdot z^2 \cdot dz
\]

\[
M_{xy} = \int \frac{1}{2} \cdot z \cdot \tau_{yz} \cdot dz = -\frac{E \cdot t^2}{12(1+\mu)} \int \frac{1}{2} \cdot z \cdot \frac{\partial^2 \omega}{\partial x \partial y} \cdot \frac{\partial^2 \omega}{\partial x \partial y} \cdot \frac{1}{2} \cdot z^2 \cdot dz
\]

\[
M_{xy} = \int \frac{1}{2} \cdot z \cdot \tau_{yz} \cdot dz = -\frac{E \cdot t^2}{12(1+\mu)} \int \frac{1}{2} \cdot z \cdot \frac{\partial^2 \omega}{\partial x \partial y} \cdot \frac{\partial^2 \omega}{\partial x \partial y} \cdot \frac{1}{2} \cdot z^2 \cdot dz
\]
Now, \( q = \int \frac{t}{2} \left( \tau_{yz} \right) \, dz \)

\[
Q_y = -\frac{E \cdot t^3}{12(1-\mu^2)} \cdot \frac{\partial}{\partial y} \left( \frac{\partial^2 \omega}{\partial x^2} \cdot \frac{\partial^2 \omega}{\partial y^2} \right)
\]

Now,
\[
\frac{\sigma_x}{t^3} = \frac{12M_{x} \cdot z}{t^3}, \quad \frac{\sigma_y}{t^3} = \frac{12M_{y} \cdot z}{t^3}, \quad \tau_{xy} = \frac{12M_{xy} \cdot z}{t^3}
\]

\[
\tau_{xz} = \frac{6Q_{x}}{t^3} = \tau_{zx}
\]

\[
\tau_{yz} = \frac{6Q_{y}}{t^3} = \tau_{zy}
\]

\[
\sigma_{z} = \left( 2q \left( \frac{1}{2} - \frac{z}{t} \right)^2 \right) \cdot (1 + \frac{z}{t})
\]

where \( q = \) total transverse load per unit area of the plate, including the surface forces and body forces in the \( z \)-direction. Considering the surface component \( z' \) on the lower face of the plate and the body force component \( z \) along their lines of action to the upper face of the plate the total surface component on the upper face becomes

\[
q = (z')_{z=-\frac{t}{2}} + (z')_{z=\frac{t}{2}} + \int_{-\frac{t}{2}}^{\frac{t}{2}} z \, dz
\]

It is the total transverse load per unit area of the plate. This force or load is considered to be positive when it acts in the positive direction \( oz \). The transmission of forces will cause some error only in the unimportant

Stress component \( \sigma_{x} \), and does not affect the other Stress component at all.

\[
q = \frac{Et^3}{12(1-\mu^2)} \cdot \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)
\]

The dimension of moments is considered as Force or Load for all practical purposes. Similarly, the dimension of shearing force is considered as Force or Load / Length.

So, the stresses on a thin plate under bending / twisting can be grouped into three classes according to the order of magnitude are (1) The transverse normal stress \( \sigma_{zz} \) is of order of magnitude of \( q \); (2) The transverse shearing stresses \( \tau_{xx} \) and \( \tau_{yy} \) are of order of \( \frac{q}{t} \); (3) The bending stresses \( \sigma_{x} \) and \( \sigma_{y} \) as well as the torsion stress \( \tau_{xz} \) are of order \( \frac{q}{t^2} \).

As for thin plate \( \frac{t}{a} \) is small in comparison with \( \frac{a}{t} \) then \( \frac{a}{t} = \) a large number. Therefore, we see that \( \sigma_{x}, \sigma_{y} \) and \( \tau_{xz} \) are much greater than \( \tau_{zz} \) and \( \tau_{yz} \) and still much greater than \( \sigma_{w} \).

So, we see the bending stress and torsion stress are acting on the thin plate. Then there is every possibility of twisting as well as bending of the plate but due to bending or shearing forces the DHS plate is uprooted along with the screw as plate is strong enough against twisting. To resist it we fix upper part of the trochanteric fracture with the lower part (i.e. to the shaft of the femur in subtrochanteric region) in three dimensional ways i.e. by additional cross screw along the conventional DHS and by replacing additional parallel screw as this cross screw will connect head of the femur to the lower part of the fracture in angular direction and it will be very much rigid because of the curved intrinsic trabecular structure of the head of the femur to fix the lower part in angular direction which will resist tendency of bending of the plate and it will be very much effective to keep the parts of the femur singly as solid one as it is a three dimensional management of fixation.

**RESULTS AND DISCUSSION**

As per study on 112, of them 39 females & 73 males, patients with trochanteric fractures had been treated by DHS (Dynamic Hip Screw) and an additional cross screw at Bhattacharyya Orthopaedic And Related Research Centre (BORRC), Narayanpur, Gopalpur, Kolkata, West Bengal (Table 1).
Now a case of Trochanteric fracture managed at Bhattacahryya Orthopaedic and Related Research Centre with DHS + Cross screw is presenting with X-rays with prognosis (Figs.11-17).

<table>
<thead>
<tr>
<th>Age-group in years</th>
<th>Number of patients</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 40</td>
<td>06</td>
<td>5.36</td>
</tr>
<tr>
<td>41 – 50</td>
<td>13</td>
<td>11.61</td>
</tr>
<tr>
<td>51 – 60</td>
<td>22</td>
<td>19.65</td>
</tr>
<tr>
<td>61 – 70</td>
<td>53</td>
<td>47.32</td>
</tr>
<tr>
<td>71 – 80</td>
<td>11</td>
<td>9.82</td>
</tr>
<tr>
<td>&gt; 80</td>
<td>07</td>
<td>6.25</td>
</tr>
<tr>
<td>Total</td>
<td>112</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 13: Antero-Posterior View after 1.5 months

Figure 14: Lateral view after 1.5 months

Figure 15: Antero-posterior view after 6 months

Figure 16: Antero-posterior view after 10 months

Figure 17: Lateral view after 10 months
DISCUSSION
In Fig.18 we see the effective forces at the trochanteric region.

We find $W =$ One-sided weight of the body acting downward through pelvis; $F =$ Reaction force at the trochanteric region i.e. on the point $T$ lying along the line of trochanter; $H_F =$ Component of Reaction force through the head of femur towards and within the pelvis and $C_F =$ Component of reaction forces away from the body. It shows forces and the reaction forces against the body generates on trochanteric-line of the femur.

Therefore, trochanteric fracture is evident with immense blow. In Figure 19 it has been shown that on the occurrence of fracture at the trochanteric region the line of action reaction force changes its direction and its point of action is shifted to the head of lesser trochanter. This phenomenon causes immediate separation of parts of the femur at the fracture region. This being taken place due to changes in lines of action and point of action of upward reaction force ($F$) against the weight of the body ($W$) acting downward but it is not acting in line with reaction force.
In inter-trochanteric fractures orthopaedicians use Dynamic Hip Screw (DHS) for fixation [Fig.20]. This type of fixation is very much responsible for absorbing translation. Translation means movement perpendicular to the plate of DHS. This fixation also helps to absorb compression and tension perpendicular to the head of femur i.e. along the shaft of the femur. Dynamicity of the hip screw accommodates the fracture face to face i.e. minimize gap between the fracture-surfaces (if any) at the time of operation. Torsion force cannot be absorbed by the DHS as it needs anti-torsion bindings. With a hope for anti-torsion binding, insertion of an additional screw parallel to DHS [Figs.21, 22, 23] was considered.

Before drawing the conclusion I have to express the conclusion of Lustenberger et al (1995) [Ref.3] in their paper that Rotational instability of a gliding implant system i.e. DHS system, is a very severe complication in the treatment of trochanteric fractures. This cross-screw will be very much rigid because of the curved intrinsic trabecular structure of the head of the femur. The angular direction of the device will resist
twisting as well as bending of the plate and thus it will stand out as a solid single unit.

To absorb $\sigma_x$, $\sigma_y$, $\tau_{xz}$, $\tau_{yz}$ along with DHS, normally used, we should put additional cross-screw to resist torsion and moments in multi-direction as DHS resist it in $xz$-direction whereas cross-screw resist it in $yz$-direction as well as $xz$-direction (both positive and negative directional moments with respect to positive direction of axes).

Only in DHS $\sigma_z$ is acting normally to the upper end of the screw forming torsional stress $\tau_{xy}$ and $\tau_{xz}$ whereas $\tau_{yz}$ is balanced by DHS itself because it has been fitted in $xz$-direction, whereas $\tau_{xy}$ cannot be absorbed by DHS but both torsional stresses are absorbed partially by the assimilation of fracture due to its roughness.

It has been deduced mathematically that the system of using cross-screw along with conventional DHS is appropriate against bending of plate with DHS. It gives better fixation with mobility as well as stability where its mechanical stability is superior. The stability of the construct has also been proved by mathematical calculations and deductions. Complications of shortening and deformity, which were obvious problems, are minimized due to consideration of three-dimensional methods of fixation of trochanteric fracture in place of two-dimensional method by DHS or by DHS with parallel screw. This construct provides good rotational bindings, providing stability, as well as early union for weight-bearing where neck-shaft angular relation is retained having a minimum deformity and limp is reduced thereby. For osteoporotic bones also, where chances of cut out are always problem, is good fixation. As the construct is stable and rigid, proved mathematically, we can provide early mobilization which allows early acceptability of daily work.

REFERENCES

Adams CI, Robinson CM, Michael C, Court-Brown CM, McQueen MM (2001): Prospective Randomized Controlled Trial of an Intramedullary Nail Versus Dynamic Screw and Plate for Intertrochanteric Fractures of the Femur; Journal of Orthopaedic Trauma; 15(6); pp.394-400.


Bhattacharyya Sailendra (2002) – Anecdotal. He introduced the cross management in 2002. Bhattacharyya Orthopaedic And Related Research Centre (BORRC), Narayanpur, Gopalpur, Kolkata – 700136, West Bengal, India


Hesse B, Gächter À (2004): Complications following the treatment of trochanteric fractures with the gamma nail; Archives of Orthopaedics and Trauma Surgery; 124(10), pp.692-698.


Research Paper


